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Concept of Set up time in Flow Shop Scheduling to optimize waiting time of jobs.

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Abstract: Flow shop scheduling refers to the execution of jobs in a pre-defined order. The set up time of machines is an important parameter while studying the waiting time of jobs. The presented model is a flow shop scheduling model in two stage where the processing times are designed in a specially structured manner. The machine set up time has also been considered separately. The algorithm has also been applied to a numerical example.

Keywords: Waiting time of jobs, Set up time, Flow shop Scheduling, Processing time.

I. INTRODUCTION

Scheduling may be defined as the problem of deciding when to execute a given set of activities, subject to chronological constraints and resources capacities, in order to optimize some function. A Flow shop problem exists when all the jobs share the same processing order on all the machines. In Flow shop, technological constraints demand that the jobs pass between the machines in the same order. Hence there is natural sequence of the machines characterized by the technological constraints for each and every job in flow shop. The flow shop contains **m** different machines arranged in series on which a set of **n** jobs are to be processed. Each of the **n** jobs requires **m** operations and each operation is to be performed on a separate machine. The flow of the work is unidirectional; thus every job must be processed through each machine in a given prescribed order. The general **n** jobs, **m** machine flow shop scheduling is quite formidable. Consider an arbitrary sequence of jobs on each machine, there are $(n!)^m$ possible schedules which poses computational difficulties. With the aim to reduce the number of possible schedules it is reasonable to assume that all machines process jobs in the same order. Efforts in the past have been made by researchers to reduce this number of feasible schedules as much as possible without compromising on optimality condition. Today's large-scale markets and instantaneous communications mean that clients expect high-quality goods and services when they require them, where they require them. Organizations, whether public or private, need to provide these products and services as effectively and efficiently as possible.

The criterion of optimality in the given flow shop scheduling problem is specified as minimization of waiting time of jobs that is defined as the sum of the times of all the jobs which was consumed in waiting for their turn on both of the machines. There are some papers in the literature of scheduling theory which consider the waiting time to be important for scheduling the jobs on the machines.

II. LITERATURE REVIEW

The Johnson's algorithm [1] is especially popular among analytical approaches that are used for solving n- jobs, 2-machines sequence problem. Ignall and Schrage [2] developed branch and bound algorithms for the permutation flow shop problem with makespan minimization. Lockett and et.al. [3], Crowin and Esogbue [4], Maggu & Dass [5] made attempts to extend the study by introducing various parameters.

Yoshida & Hitomi [6] solved two stage production scheduling, the set up time being separated from processing time. Solution methods for flow shop scheduling range from heuristics developed by Singh T.P. [7], Rajendran and Chaudhuri [10]. Singh T.P., Gupta D. [11] studied the problem related with group job restrictions in a flow shop which involves independent set- up time and transportation time. Singh Vijay [15] put his efforts to study three machine flow shop scheduling problems with total rental cost.

Further Gupta D. [16] studied minimization of Rental Cost in Two Stage Flow Shop Scheduling Problem, in which Setup Time was separated from Processing Time and each associated with probabilities including Job Block Criteria. Gupta D & et.al.[8], [9] studied optimal two and three stage open shop specially structured scheduling to minimize the rental cost, processing time associated with probabilities including transportation time. Gupta D. & Goyal B.[17], [18] studied the concept of minimizing waiting time of jobs in which processing times are associated with probabilities.

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The problem discussed here has significant use of theoretical results in process industries or in the situations when the objective is to minimize the total waiting time of jobs. The paper discussed here is an extension made by Gupta D. & Goyal B. [17] in the sense that we have taken into consideration the set up time of machine separated from processing time.

III. PRACTICAL SITUATION

Manufacturing units/industries play an important role in the economic development of a country. Flow shop scheduling occurs in various offices, service stations, banks, airports etc. In our day to day working in factories and industrial units different jobs are processed on various machines.

In textile industry different types of fabric is produced using different types of yarn. Here, the maximum equivalent time taken in dying of yarn on first machine is always less than or equal to the minimum equivalent time taken in weaving of yarn on the second machine. The idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's view point when he has minimum time contract with a commercial party to complete the jobs

NOTATIONS

S_j : Sequence obtained by applying the algorithm proposed.
P_j : Time for processing of i^{th} job on machine P.
Q_j : Time for processing of i^{th} job on machine Q.
P_j' : Equivalent time for processing of i^{th} job on machine P.
Q_j' : Equivalent time for processing of i^{th} job on machine Q.
s_j : Set up time of i^{th} job on machine P.
t_j : Set up time of i^{th} job on machine Q.
C_{aQ} : The completion time of job a on machine Q.
W_β : Waiting time of job β.
T_w : Total waiting time of all the jobs.

IV. PROBLEM FORMULATION

Assume that two machines P and Q are processing n jobs in the order P Q. P_i and Q_i are the respective processing times and s_i and t_i are the respective set up times of the i^{th} job on machines P & Q. Our intention is to find an optimal sequence $\{S_k\}$ of jobs minimizing the total waiting time of all jobs.

Equivalent processing times of i^{th} job on machine P & Q are defined as

$P_i' = P_i - t_i$, $Q_i' = Q_i - s_i$ Satisfying processing times structural relationship Max $P_i' \leq$ Min Q_i'

TABLE 1: MATRIX FORM OF THE MATHEMATICAL MODEL OF THE PROBLEM

Job	Machine P		Machine Q	
I	P_i	s_i	Q_i	t_i
1.	P_1	s_1	Q_1	t_1
2.	P_2	s_2	Q_2	t_2
3.	P_3	s_3	Q_3	t_3
.	.	.	.	.
	.	.	.	.
n.	P_n	s_n	Q_n	t_n

ASSUMPTIONS

In the given flow shop scheduling the following assumptions are made

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1) There are *n* number of jobs (I) and two machines (P & Q).
2) The order of sequence of operations in all machines is the same.
3) Jobs are independent to each other.
4) Machines bream down interval, transportation time is not considered for calculating waiting time.
5) Pre- emption is not allowed i.e. jobs are not being split clearly, once a job is started on a machine, the process on that machine can't be stopped unless the job is completed.

Lemma 1. Let two machines P, Q are processing n jobs in order $P\,Q$ with no passing allowed. Let P_i and Q_i are the processing times of job i ($i = 1,2,3, \ldots, n$) on each machine respectively assuming their respective set up times s_i and t_i. Equivalent processing times of i^{th} job on machine $P\,\&\,Q$ are defined as $P_i' = P_i - t_i \quad Q_i' = Q_i - s_i$ satisfying processing times structural relationship Max $P_i' \leq$ Min Q_i' then for the n job sequence S: $\beta_1, \beta_2, \beta_3, \ldots \ldots \ldots \beta_n$

$$C_{\beta_n Q} = P'_{\beta_1} + Q'_{\beta_1} + Q'_{\beta_2} \ldots + Q'_{\beta_n}$$

Where C_{aQ} is the completion time of job a on machine Q.

Proof. Applying mathematical Induction hypothesis on n:

Let the statement $S(n): C_{\beta_n Q} = P'_{\beta_1} + Q'_{\beta_1} + Q'_{\beta_2} \ldots + Q'_{\beta_n}$

$$C_{\beta_1 P} = P'_{\beta_1}$$
$$C_{\beta_1 Q} = P'_{\beta_1} + Q'_{\beta_1}$$

Hence for n= 1 the statement $S(1)$ is true.
Let for n= m, the statement $S(m)$ be true, i.e.,

$$C_{\beta_m Q} = P'_{\beta_1} + Q'_{\beta_1} + Q'_{\beta_2} \ldots + Q'_{\beta_m}$$

Now,
$$C_{\beta_{m+1} Q} = \mathrm{Max}\left(C_{\beta_{m+1} P}, C_{\beta_m Q}\right) + Q'_{\beta_{m+1}}$$

As Max $P_i' \leq$ Min Q_i'

Hence
$$C_{\beta_{m+1} Q} = P'_{\beta_1} + Q'_{\beta_1} + Q'_{\beta_2} \ldots + Q'_{\beta_m} + Q'_{\beta_{m+1}}$$

Hence for $n = m + 1$ the statement $S(m + 1)$ holds true. Since $S(n)$ is true for $n = 1, n = m$,

$n = m + 1$, and m being arbitrary. Hence $S(n): C_{\beta_n Q} = P'_{\beta_1} + Q'_{\beta_1} + Q'_{\beta_2} \ldots + Q'_{\beta_n}$ is true.

Lemma 2. With the same notations as that of Lemma1, for n- job sequence S: $\beta_1, \beta_2, \beta_3, \ldots, \beta_m, \ldots, \beta_n$

$$W_{\beta_1} = 0$$
$$W_{\beta_m} = P'_{\beta_1} + \sum_{r=1}^{m-1} x_{\beta_r} - P'_{\beta_m}$$

Where W_{β_m} is the waiting time of job β_m for the sequence $(\beta_1, \beta_2, \beta_3, \ldots \ldots \ldots \beta_n)$ and

$$x_{\beta_r} = Q'_{\beta_r} - P'_{\beta_r}, \quad \beta_r \in (1, 2, 3, \ldots, n)$$

Proof. $W_{\beta_1} = 0$

$$W_{\beta_m} = \mathrm{Max}\left(C_{\beta_m P}, C_{\beta_{m-1} Q}\right) - C_{\beta_m P}$$

$$= P'_{\beta_1} + Q'_{\beta_1} + Q'_{\beta_2} \ldots + Q'_{\beta_{m-1}} - P'_{\beta_1} - P'_{\beta_2} \ldots - P'_{\beta_m}$$

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$$= P'_{\beta_1} + \sum_{r=1}^{m-1} (Q'_{\beta_r} - P'_{\beta_r}) - P'_{\beta_m}$$

$$= P'_{\beta_1} + \sum_{r=1}^{m-1} (x_{\beta_r}) - P'_{\beta_m}$$

Theorem 1. Let two machines P, Q are processing n jobs in order $P\,Q$ with no passing allowed. Let P_i and Q_i are the processing times of job i ($i = 1,2,3, \dots, n$) on each machine respectively assuming their respective set up times s_i and t_i. Equivalent processing times are defined as $P'_i = P_i - t_i$ $Q'_i = Q_i - s_i$ satisfying processing times structural relationship $\text{Max } P'_i \le \text{Min } Q'_i$ then for any n job sequence S: $\beta_1, \beta_2, \beta_3, \dots \dots \beta_n$ the total waiting time T_w (say)

$$T_w = nP'_{\beta_1} + \sum_{r=1}^{n-1} z_{\beta_r} - \sum_{i=1}^{n} P'_i$$

$$z_{\beta_r} = (n-r)x_{\beta_r} \; ; \beta_r \epsilon (1, 2, 3, \dots, n)$$

Proof. From Lemma 2 we have

$$W_{\beta_1} = 0$$

For $m = 2$,

$$W_{\beta_2} = P'_{\beta_1} + \sum_{r=1}^{1} x_{\beta_r} - P'_{\beta_2}$$

For $m = 3$,

$$W_{\beta_3} = P'_{\beta_1} + \sum_{r=1}^{2} x_{\beta_r} - P'_{\beta_3}$$

Continuing in this way

For $m = n$,

$$W_{\beta_n} = P'_{\beta_1} + \sum_{r=1}^{n-1} x_{\beta_r} - P'_{\beta_n}$$

Hence total waiting time

$$T_w = \sum_{i=1}^{n} W_{\beta_i}$$

$$T_w = nP'_{\beta_1} + \sum_{r=1}^{n-1} z_{\beta_r} - \sum_{i=1}^{n} P'_i$$

Where $z_{\beta_r} = (n-r)x_{\beta_r}$

ALGORITHM

Step 1: Equivalent processing times P'_i and Q'_i on machine P & Q respectively be calculated in first step as defined in the lemma 1.

Step 2: Calculate the entries for the following table

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TABLE 2

Job	Machine P	Machine Q		$z_{ir} = (n-r)x_i$				
I	P'_i	Q'_i	x_i	r = 1	r = 2	r = 3	...	r = n-1
1.	P'_1	Q'_1	x_1	z_{11}	z_{12}	z_{13}	⋯	$z_{1\,n-1}$
2.	P'_2	Q'_2	x_2	z_{21}	z_{22}	z_{23}	⋯	$z_{2\,n-1}$
3.	P'_3	Q'_3	x_3	z_{31}	z_{32}	z_{33}	⋯	$z_{3\,n-1}$
.	.	.	.	.	.	.	.	.
n.	P'_n	Q'_n	x_n	z_{n1}	z_{n2}	z_{n3}	⋯	$z_{n\,n-1}$

Step 3: Assemble the jobs in increasing order of x_i.

Assuming the sequence found be $(\beta_1, \beta_2, \beta_3, \dots \dots \dots \beta_n)$

Step 4: Locate $\min\{P'_i\}$

For the following two possibilities
$P'_{\beta_1} = \min\{P'_i\}$ Schedule according to step 3 is the required optimal sequence
$P'_{\beta_1} \neq \min\{P'_i\}$ move on to step 5

Step 5: Consider the different sequence of jobs $S_1, S_2, S_3, \dots \dots, S_n$. Where S_1 is the sequence obtained in step 3, Sequence $S_j (j = 2,3, \dots \dots, n)$ can be obtained by placing j^{th} job in the sequence S_1 to the first position and rest of the sequence remaining same.

Step 6: Compute the total waiting time T_w for all the sequences $S_1, S_2, S_3, \dots \dots, S_n$ using the following formula:

$$T_w = nP'_b + \sum_{r=1}^{n-1} z_{ar} - \sum_{i=1}^{n} P'_i$$

P'_b = Equivalent processing time of the first job on machine P in sequence S_j

$z_{ar} = (n-r)x_{ar}$; $a = \beta_1, \beta_2, \beta_3, \dots \dots \dots \beta_n$

The sequence with minimum total waiting time is the required optimal sequence.

V. NUMERICAL ILLUSTRATION

Assume 5 jobs 1, 2, 3, 4, 5 has to be processed on two machines P & Q with processing times P_i and Q_i and set up times s_i and t_i respectively

TABLE 3: PROCESSING TIME MATRIX

Job	Machine P		Machine Q	
I	P_i	s_i	Q_i	t_i
1.	5	1	9	3
2.	7	3	8	2
3.	4	4	10	1
4.	2	2	7	1
5.	6	1	8	4

Our objective is to obtain optimal string, minimizing the total waiting time for the jobs.
Solution

As per step 1- Equivalent processing time P_i' & Q_i' on machine P & Q given in the following table

`TABLE 4: EQUIVALENT PROCESSING TIME MATRIX

Job	**Machine P**	**Machine Q**
I	$\mathbf{P_i'}$	$\mathbf{Q_i'}$
1.	2	8
2.	5	5
3.	3	6
4.	1	5
5.	2	7

Max $P_i' = 5 \leq$ Min $Q_i' = 5$
As per step 2- Obtaining the values for TABLE 2

TABLE 5

Job	**Machine P**	**Machine Q**		$\mathbf{z_{ir} = (5-r)x_i}$			
I	$\mathbf{P_i'}$	$\mathbf{Q_i'}$	$\mathbf{x_i = Q_i' - P_i'}$	**r = 1**	**r = 2**	**r = 3**	**r = 4**
1.	2	8	6	24	18	12	6
2.	5	5	0	0	0	0	0
3.	3	6	3	12	9	6	3
4.	1	5	4	16	12	8	4
5.	2	7	5	20	15	10	5

As per step 3-.The sequence thus found be 2, 3, 4, 5, 1.

As per step 4- $\text{Min}\{P_i'\} = 1 \neq P_1'$

As per step 5- Different sequence of jobs can be considered as:
$S_1: 2, 3, 4, 5, 1$; $S_2: 3, 2, 4, 5, 1$; $S_3: 4, 2, 3, 5, 1$; $S_4: 5, 2, 3, 4, 1$; $S_5: 1, 2, 3, 4, 5$

As per step 6- The total waiting time for the sequences obtained in step 5 can be calculated

Here, $\sum_{i=1}^{5} P_I' = 13$

For the sequence $S_1: 2, 3, 4, 5, 1$
Total waiting time $T_w = 34$
For the sequence $S_2: 3, 2, 4, 5, 1$
Total waiting time $T_w = 27$
For the sequence $S_3: 4, 2, 3, 5, 1$
Total waiting time $T_w = 19$
For the sequence $S_4: 5, 2, 3, 4, 1$
Total waiting time $T_w = 27$
For the sequence $S_5: 1, 2, 3, 4, 5$
Total waiting time $T_w = 31$

Hence schedule $S_3: 4, 2, 3, 5, 1$ is the required schedule with minimum total waiting time.

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VI. CONCLUSION

The present study deals with the flow shop scheduling problem with the main idea to minimize the total waiting time of jobs. However it may increase the other costs lime machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's view point when he has minimum time contract with a commercial party to complete the jobs. The work can be extended by introducing various parameters like transportation time, break down interval etc.

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