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A Ranking Function Approach for Solving Sequencing Problems under Trapezoidal Fuzzy Numbers

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Abstract: The job sequencing problem is an important branch of operations research where the order of processing jobs affects system performance. Due to vagueness and uncertainty in crisp theory sequencing problems are taken in which processing times are taken as fuzzy numbers. In this paper processing times are taken as trapezoidal fuzzy numbers and a new ranking function is proposed for determination of the optimal sequence of jobs, aiming to minimize the total elapsed time. Numerical example is given for demonstrating the proposed ranking function.

Keywords: Sequencing problem, trapezoidal fuzzy numbers, ranking function, optimal sequence, total elapsed time.

I. INTRODUCTION

In sequencing problem, the challenge lies in finding an optimal sequence that minimizes the total elapsed time and tardiness. However, in real-world processing times are not always deterministic. This motivated the inculcation of fuzzy set theory in sequencing problems. Zadeh (1965, 1967) gave the concept of fuzzy set. He introduced this concept by taking into consideration the imprecisely defined classes, for example “the class of beautiful woman” or “the class of tall men”. Definition of fuzzy set and their properties are given. Fuzzy set theory permits the gradual assessment of the membership of elements in a set which is described in the internal [0,1]. It can be used in a wide range of domains where information is in complete and imprecise. Relations and ordering were given by taking equivalence relation as a base. Bellman and Zadeh (1970) used fuzzy theory in decision making in which goals and constraints were fuzzy in nature. However, the system is not. The concept used was illustrated with the help of examples. In decision making problem there are many imprecise concepts which could not be defined uniquely and this problem was best solved by with fuzzy theory. Jain (1976) was first who proposed ranking of fuzzy numbers and used in fuzzy decision making where vague quantities are represented as fuzzy sets. Zadeh (1978) gave the theory of fuzzy sets and defined it as possibility theory rather than probability. In this theory fuzzy variable is connected with possibility distribution. Jain (1978) proposed method to solve qualitative data quantitatively with the help of fuzzy sets. In the method presented by Jain final rating of each alternative is calculated. Arithmetic operations on fuzzy quantities were given and the method used was explained with the help of numerical example. Yager (1978) illustrated the ordering of fuzzy numbers. For this procedure was given in which regular fuzzy numbers are mapped into real numbers came out with meaningful ordering of fuzzy numbers. Dubois and Prade (1976, 1978) had defined fuzzy numbers as a fuzzy subset of the real line. It is a quantity which is imprecise rather than exact and is a subset of the real line whose membership vales cluster around the value called mean value. Dubois and prade in this paper extended usual operations of real numbers to fuzzy numbers with the help of fuzzification principle. Yager (1980, 1981) introduced a class of connectives, intersection and union for fuzzy sets. The properties of this class are also studied by comparing with union and intersection of ordinary sets. He also defined a function which helped in the ordering of fuzzy subsets of the unit interval and its properties. Chen et al. (1984) used fuzzy approach in multiobjective scheduling for finding optimal processing sequence.

Preliminaries

Definition 1

A fuzzy number $\tilde{A} = (a, b, c, d)$, is said to be a trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(d-x)}{(d-c)}, & c \leq x \leq d \\ 0, & otherwise \end{cases}$$

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where $a,b,c,d \in R$

Definition 2(Yager's ranking)

The ranking function of trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ is defined as

$$R(\tilde{A}) = \frac{2a + 2b + c + d}{6}$$

Definition 3(Proposed Ranking)

Let a trapezoidal fuzzy number be $\tilde{A} = (a,b,c,d)$, then the proposed ranking function is defined as $R(\tilde{A}) = \alpha \frac{a+b+c+d}{4} + (1-\alpha)\frac{b+c}{2}$, where $\alpha \in [0,1]$ is a weighting factor balancing between overall spread and core interval. If $\alpha = 0.5$, the ranking equally considers both average spread and the most certain interval.

Algorithm to solve fuzzy job sequencing problems

We have many types of sequencing problems. In every problem objective is to minimize the total elapsed time and waiting time for the jobs. Here, let us consider the problem consisting of n-jobs processed through 2-machines. In this method, processing time of machines is taken as trapezoidal fuzzy numbers and thus sequencing problem can be written as follows:

Jobs→ Machines↓	J_1	J_2	…….	J_n
M_1	$\tilde{t}_{11}$	$\tilde{t}_{12}$	………	$\tilde{t}_{1n}$
M_2	$\tilde{t}_{21}$	$\tilde{t}_{22}$	…….	$\tilde{t}_{2n}$

Here $\tilde{t}_{ij}$ is trapezoidal fuzzy numbers which denotes time duration taken by i^{th} job on j^{th} machine.

Stepwise algorithm is as follows:

Step1. Start

Step 2. Find ranking function of all processing times t_{ij}.

Step 3. Add processing times of two machines corresponding to each job.

Step 4. From these numbers, find the smallest sum and if it corresponds to first machine then place that job in the first available position in the sequence.

Step 5. If this is for second machine, then write that job in the last available position in the sequence. In case of tie: if it is for same machine then select lowest subscript job for machine 1 and largest subscript for machine 2.

Step 6. Do not consider jobs that have assigned position.

Step 7. Repeat these steps until all the jobs are included in the sequence.

Step 8. Calculate overall elapsed time and idle time for both machines.

Step 9. End.

Numerical Illustration

There are five tasks A, B, C, D and E which must go through two machines M_1 and M_2. The fuzzy processing times for all the tasks on two machines are given below:

Jobs→ Machines↓	A	B	C	D	E
M_1	(2,4,6,8)	(1,2,5,7)	(1,3,4,9)	(2,4,4,7)	(1,3,5,6)
M_2	(0,4,8,10)	(3,6,9,12)	(5,7,8,9)	(2,4,7,10)	(2,3,5,8)

Solution:

Step1:

Using proposed ranking do defuzzification of the processing times.

Jobs→ Machines↓	A	B	C	D	E
M_1	5	3.625	3.875	4.125	3.625
M_2	5.75	6.25	6.375	5.625	4.25

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Step2:
Add processing times of both the machines.

Jobs→ Machines↓	A	B	C	D	E
Sum	10.75	9.875	10.25	9.75	7.875

Job sequence of this table is

E	-	-	-	-

Step3:
Reduced table is:

Jobs→ Machines↓	A	B	C	D
M_1	5	3.625	3.875	4.125
M_2	5.75	6.25	6.375	5.625

Add processing times of both the machines.

Jobs→ Machines↓	A	B	C	D
Sum	10.75	9.875	10.25	9.75

Job sequence of this reduced table is

E	D	-	-	-

Step4:
Reduced table is:

Jobs→ Machines↓	A	B	C
M_1	5	3.625	3.875
M_2	5.75	6.25	6.375

Add processing times of both the machines.

Jobs→ Machines↓	A	B	C
Sum	10.75	9.875	10.25

Job sequence of this reduced table is

E	D	B	-	-

Step5:
Reduced table is:

Jobs→ Machines↓	A	C
M_1	5	3.875
M_2	5.75	6.375

Add processing times of both the machines.

Jobs→ Machines↓	A	C
Sum	10.75	10.25

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Job sequence of this reduced table is

E	D	B	C	-

Final Sequence is

E	D	B	C	A

The minimum total elapsed time is given by:

Job Sequence	M_1		M_2	
	Time in	Time out	Time in	Time out
E	0	(1,3,5,6)	(1,3,5,6)	(3,9,10,14)
D	(1,3,5,6)	(3,7,9,13)	(3,9,10,14)	(5,13,17,24)
B	(3,7,9,13)	(4,9,14,20)	(5,13,17,24)	(8,19,26,36)
C	(4,9,14,20)	(5,12,16,29)	(8,19,26,36)	(13,16,34,45)
A	(5,12,16,29)	(7,16,22,37)	(13,16,34,45)	(13,20,42,55)

and total elapsed time obtained is (13,20,42,55) ≈ 31.75 hours.

II. CONCLUSION

In this paper, a ranking function approach was proposed to address sequencing problems with processing times represented by trapezoidal fuzzy numbers. The developed ranking function effectively transforms trapezoidal fuzzy numbers into comparable crisp values, enabling the application of classical sequencing rules. Future research may extend this framework to multi-machine, multi-objective, or stochastic sequencing environments.

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