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Abaoub-Shkheam Decomposition Method for Solving Second order Non-Linear Ordinary Differential Equations

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Abstract: The Abaoub-Shkheam Decomposition Method (QDM) is employed in this paper to solve nonlinear initial value problems of second order. Adomian polynomial is used to decompose nonlinear functions that exist in a given equation. We are using this method (QDM) to find the exact solution of different types of non-linear ordinary differential equations, which is based on the Abaoub Shkheam transform method (QTM) and the Adomian Decomposition Method (ADM). An example is provided to demonstrate the efficacy of this approach.

Keywords: Ordinary differential equations, Abaoub Shkheam transform, Adomian Decomposition Methodise.

I. INTRODUCTION

Linear and nonlinear differential equations are used to simulate the majority of issues in the natural and engineering sciences, including fluid flow and heat transport in region. A wide class of ordinary, partial, deterministic, and stochastic differential equations, both linear and nonlinear, can be solved [1–11] quickly, simply, and precisely using the decomposition Method.

II. ABAOUB SHKHEAM TRANSFORM METHOD

Ali Abaoub, and Abejela Shkheam[12,13] gave some properties and applications of Abaoub Shkheam transform. Laplace and Sumudu transform methods are related to the Abaoub Shkheam transform method. Some fundamental characteristics, like the first shift, change of scale transform of derivative and integral of Abaoub Shkheam transform. Application of Abaoub Shkheam transform in particular to the partial differential equation [14]. The Adomian Decomposition Method (ADM) is a brand-new, extremely powerful method that Adomian [15], first presented in the early 1980s for solving a wide range of equations, including integral, differential, partial differential, and linear and non-linear algebraic equations [16–25]. The solution series has shown to rapidly converge using this strategy. The non-linear term is broken down into a set of specialized polynomials known as Adomian's polynomials in order for it to function.

Let the real function $f(x) > 0$ and $f(x) = 0$ for $x < 0$ is piecewise continuous, exponential order and define by

$$A = \left\{ f(x) : \exists\, M, t_1, t_2 > 0, |f(x)| < M\, e^{\frac{x}{t_i}}, if\ x \in (-1)^j \times [0, \infty) \right\};$$

The Abaoub Shkheam transform of the function $f(x) > 0$ and $f(x) = 0$ for $x < 0$ is given by

$$Q[\, f(x)] = \int_0^\infty f(ut)\, e^{-\frac{t}{s}} dt = T(u, s).$$

Where s and u are the transform variables.

III. ABAOUB SHKHEAM DECOMPOSITION METHOD

The general form of non-linear ordinary differential equation of the second order is given by :

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$$L[y(x)] + R[y(x)] + N[y(x)] = f(x), \quad (1)$$

where $L = \frac{d^2}{dx^2}$is linear second order differential operator, R is the is remainder of the differential operator, N represents the general non-linear differential operator, and $f(x)$ is the non-homogeneous term. with the initial conditions:

$$y(0) = g(x), y'(0) = h(x). \quad (2)$$

The method entails utilising the Abaoub Shkheam transformation (denoted as Q) on both sides of equation (1).

$$Q\left[\frac{d^2}{dx^2}y\right] + Q[R(y) + N(y)] = Q[f(x)]. \quad (3)$$

Applying the Abaoub Shkheam transform formulae to equation (3) yield

$$\frac{1}{u^2s^2}Y(u,s) - \frac{1}{u^2s}y(0) - \frac{1}{u}y'(0) + Q[R(y) + N(y)] = F(x), \quad (4)$$

Substituting the initial conditions in equation (2) to equation (4), this gives

$$Y(u,s) = sg(x) + us^2h(x) - u^2s^2[Q\{R[y(x)] + N[y(x)]\}]. \quad (5)$$

Taking the inverse of Q-transform to the equation (3.1.7), we get:

$$y(x) = V(x) - Q^{-1}[(us)^2Q\{R[y(x)] + N(y)\}], \quad (6)$$

where $V(x)$ represents the term arising from the source term and the prescribed the initial conditions.

The solution is represented as an infinite series using the Abaoub Shkheam transform decomposition technique.

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \quad (7)$$

where the computation of the terms $y_n(x)$ is to be done recursively. Additionally, the nonlinear operator $N(y)$ can be decomposed as follows:

$$N(y) = \sum_{n=0}^{\infty} A_n(y), \quad (8)$$

Where $A_n = (y_1, y_2, y_{3,} \dots yy_n)$is the so-called Adomian polynomial,which represent the non-linear term, and it can be calculated by the following formula:

$$A_n(y) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i y_i\right)\right]_{\lambda=0}, n = 0,1,2, \dots . \quad (9)$$

Equations (7) and (87) can be substituted into equation (6) obtain

$$\sum_{n=0}^{\infty} y_n(x) = V(x) - Q^{-1}\left[(us)^2Q\left\{R\sum_{n=0}^{\infty} y_n(x) + \sum_{n=0}^{\infty} A_n(y)\right\}\right]. \quad (10)$$

The iterative process that results from equating both sides of equation (10) is as follows:

$$y_0(x) = V(x),$$

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$$y_1(X) = -Q^{-1}[(us)^2 Q\{Ry_0(x) + A_0(y)\}],$$
$$y_2(X) = -Q^{-1}[(us)^2 Q\{Ry_1(x) + A_1(y)\}],$$
$$y_3(x) = -Q^{-1}[(us)^2 Q\{Ry_2(x) + A_2(y)\}],$$

In general,

$$y_{n+1}(x) = -Q^{-1}\{(us)^2\, Q\{Ry_n(x) + A_n(y)\}\}, n \geq 0.$$

By using the above equations, we get the exact solution which given by the infinite series (7).

IV. APPLICATIONS OF ABAOUB SHKHEAM DECOMPOSITION METHOD

1. Consider the following first order non-linear ordinary differential equation:

$$\frac{dy}{dx} = y^2(x) + 1, \tag{11}$$

with initial condition:

$$y(0) = 0. \tag{12}$$

Applying the Abaoub Shkheam transform on both sides of Eq.(11), and using the initial condition (12)

$$Y(u,s) = us^2 + usQ[y^2(x)].$$

Taking the inverse Abaoub-Shkheam transform

$$\mathrm{y}(x) = x + Q^{-1}\left[usQ[y^2(x)]\right],$$

we can express this solution as eq.(7), we obtain

$$\sum_{n=0}^{\infty} y_n(x) = x + Q^{-1}\left[usQ\left\{\sum_{n=0}^{\infty} A_n(y)\right\}\right],$$

Where the nonlinear term operator $N(y)$ are decomposed from Adomian polynomials.

The first few Adomian polynomials for $N(y)$ is given by

$$y_0(x) = x,$$

$$y_1(x) = Q^{-1}[us\, Q\{A_0(y)\}] = \frac{1}{3}x^3,$$

$$y_1(x) = \mathrm{Q}^{-1}[\mathrm{us\ Q}\{A_1(y)\}] = Q^{-1}[2u^2s^3] = \frac{1}{3}x^3,$$
$$\vdots$$

Then the exact solution of the unknown function $y(x)$is given by:

$$y(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots = \tan x\,.$$

2. Consider the following first order nonlinear ordinary differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y^2(x) + \cos x = 1, \tag{13}$$

with initial condition:

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$$y(0) = 1, \qquad y'(0) = 0. \tag{14}$$

Applying the Abaoub Shkheam transform on both sides of Eq.(13), and using the initial condition (14)

$$Y(u,s) = u^2 s^3 + \frac{s}{1+u^2s^2} - (us)^2 Q\left[\left(\frac{dy}{dx}\right)^2 + y^2(x)\right]. \tag{15}$$

Taking the inverse Q-Trans form of equation (15), we have:

$$y(x) = \frac{x^2}{2} + \cos x - Q^{-1}\left[(us)^2 Q\left[\left(\frac{dy}{dx}\right)^2 + y^2(x)\right]\right] \tag{16}$$

we can express this solution as eq.(7), we obtain

$$\sum_{n=0}^{\infty} y_n(x) = \frac{x^2}{2} + \cos x - Q^{-1}\left[(us)^2 Q \sum_{n=0}^{\infty}\{A_n(y) + B_n(y)\}\right], \tag{17}$$

where A_n and B_n are Adomian polynomials of the non-linear term $\left(\frac{dy}{dx}\right)^2$ and $y^2(x)$ respectively. Comparing both sides by equation(17),we can drive the general recursive as follows:

$$y_0(x) = \frac{x^2}{2} + \cos x,$$

$$y_1(x) = -Q^{-1}\left[(us)^2 Q[A_0(y) + B_0(y)]\right] = -\frac{x^2}{2},$$

$$y_2(x) = -Q^{-1}\left[(us)^2 Q[A_1(y) + B_1(y)]\right] = -Q^{-1}\left[u^2s^2 Q[0]\right] = 0,$$

$$y_3(x) = -Q^{-1}\left[(us)^2 Q[A_2(y) + B_2(y)]\right],$$

Similarly,

$$y_3(x) = 0, \qquad n \geq 0.$$

Then

$$y(x) = \cos x.$$

V. CONCLUSION

This paper presents a novel method for solving nonlinear ordinary differential equations by combining the Abaoub-Shkheam transform with the Adomian Decomposition Method (Q.A.D.M). This approach effectively simplifies the solution process for these equations, demonstrating that Q.A.D.M is a robust mathematical tool for addressing a wide range of nonlinear differential equations and obtaining exact solutions.

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