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Application of Geometric and Fuzzy Geometric Programming in A Probabilistic Model

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Abstract: In this paper a probabilistic inventory model with deterministic and stochastic environments is discussed. We analyze the model by geometric programming and fuzzy geometric programming techniques. During the entire discussion the model is discussed under Uniform and exponential lead time demands. Finally it is concluded that fuzzy geometric programming gives us more optimized result than geometric Programming Technique.

Keywords: Geometric Programming, Fuzzy Geometric Programming, Stochastic Environment.

1. INTRODUCTION

In most of the existing inventory models, it is assumed that the inventory parameters, objective goals and constraint goals are deterministic and fixed. But, if we think of their practical meaning, they are uncertain, either random or imprecise. When some or all parameters of an optimization problem are described by random variables, the problem is called stochastic or probabilistic programming problem. In a stochastic programming problem, the uncertainties in the parameters are represented by probability distributions. This distribution is estimated on the basis of the available observed random data. In 1965, the first publication in fuzzy set theory by Zadeh (1965) showed the intention to accommodate uncertainty in the non-stochastic sense rather than the presence of random variables. Bellman and Zadeh (1970) first introduced fuzzy set theory in decision-making processes. Later, Tanaka, et-al. (1974) considered the objectives as fuzzy goals over the α-cuts of a fuzzy constraint set and Zimmermann (1976) showed that the classical algorithms could be used to solve a fuzzy linear programming problem. Fuzzy mathematical programming has been applied to several fields like project network, reliability optimization, transportation, media selection for advertising; air pollution regulation etc. problems formulated in fuzzy environments. Detail literature on fuzzy linear and non-linear programming with application is available in two well-known books of Lie and Hwang (1992, 1994). Walter (1992) discussed the single period inventory problem with uniform demand. In inventory problem, fuzzy set theory has not been much used. Park (1987) examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with cost data. Roy and Maiti (1995, 1998) solved the classical EOQ models in fuzzy environment with fuzzy objective goal and constraint by fuzzy non-linear programming and fuzzy additive goal programming techniques.
During last several years, fuzzy GP has received rapid development in the theory and application (2007). Many scholars have done work in this area; they come from China, India, Iran, China Taiwan, Belgium, Canada, Germany (1990), Egypt, Cuba, etc. In 2001, B. Y. Cao published the first monograph of fuzzy GP as applied optimization series (Vol.76), Fuzzy Geometric Programming, by Kluwer Academy Publishing (the present Springer) (2002), the book gives a detailed exposition to theory and application of fuzzy GP. Several fuzzy models for single-period inventory problem were discussed by Lushu, Kabadi and Nair (2002). Mahapatra, G.S. and Roy, T.K. (2006) used General Fuzy Programming technique on a reliability optimization model. Cao (1993) and his recent book (2002) discussed fuzzy geometric programming with zero degree of difficulty. Das et. al. (2000) developed a multi-item inventory model with quantity dependent inventory costs and demand dependent unit cost under imprecise objective function and constraint and solved by GP technique. Recently Mondal et. al. (2005) developed a multi-objective inventory model and solved it by GP method. A multi-objective fuzzy economic production quantity model is solved using GP approach by Islam and Roy (2004). Islam and Roy (2007) solved another fuzzy economic production quantity model under space constraint by GP method.

In this paper we consider a Stochastic Inventory model with deterministic and fuzzy constraints as well. It is modeled with Uniform and Exponential lead time demand. The model is analyzed by means of Geometric Programming and also by Fuzzy Geometric Programming Techniques. Finally the numerical illustration established the fact that using Fuzzy Geometric Programming more optimized result is obtained than Geometric Programming Technique.

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2. MATHEMATICAL MODEL

Multi-item Paknejad et al.'s model (1995) along with the notations and some assumptions will be taken into account throughout the paper. Each lot contains a random number of defectives following binomial distribution. After the arrival purchaser examines the entire lot. An order of size Q is placed as soon as the inventory position reaches the reorder point s. the shortages are allowed and completely backordered. Lead-time is constant and probability distribution of lead-time demand is known. Thus a quality adjusted lot-sizing model is formed as:

EC ($Q_1,Q_2,\ldots Q_n, s_1,s_2,\ldots s_n$) = setup cost + non-defective item holding cost + stock out cost + defective item holding cost + inspecting cost

$$\sum_{i=1}^{n}\left(\frac{D_iK_i}{Q_i(1-\theta_i)}+h_i(s_i-\mu_i+\frac{1}{2}(Q_i(1-\theta_i)+\theta_i))+\frac{D_i\pi_i\bar{b}_i(s)}{Q_i(1-\theta_i)}+h_i'\theta_i(Q_i-1)+\frac{D_iv_i}{1-\theta_i}\right)$$

Where, (for the i^{th} item)

D_i = expected demand per year

Q_i = lot size

s_i = reorder point

K_i = setup cost

θ_i = defective rate in a lot of size Q, $0 \le \theta \le 1$

h_i = nondefective holding cost per unit per year

h_i' = defective holding cost per unit per year

π_i = shortage cost per unit short

v_i = cost of inspecting a single item in each lot

μ_i = expected demand during lead time

p_i = purchasing price of each product

B = total budget, F = total Floor Space area

$\bar{b}_i(s_i)$ = the expected demand short at the end of the cycle

$$\bar{b}_i(s)=\int_{s_i}^{\infty}(x-s_i)f_i(x)dx$$

Where, $f_i(x)$ is the density function of lead-time demand.

EC ($Q_1,Q_2,\ldots Q_n, s_1,s_2,\ldots s_n$) = expected annual cost given that a lot size Q is ordered.

2.1 A Multi Objective Multi-Item Stochastic Inventory Model

It is the constrained stochastic model, which minimizes the expected annual cost, and it can be stated as item wise multi objective model.

A. Model with Deterministic Budget and Floor Space

Min EC_i ($Q_1,Q_2,\ldots Q_n, s_1,s_2,\ldots s_n$) =

$$\left(\frac{D_iK_i}{Q_i(1-\theta_i)}+h_i(s_i-\mu_i+\frac{1}{2}(Q_i(1-\theta_i)+\theta_i))+\frac{D_i\pi_i\bar{b}_i(s)}{Q_i(1-\theta_i)}+h_i'\theta_i(Q_i-1)+\frac{D_iv_i}{1-\theta_i}\right) \quad \ldots..(2.1)$$

subject to the constraints

$$\sum_{i=1}^{n}p_iQ_i \le B$$

$$\sum_{i=1}^{n}f_iQ_i \le F$$

$Q_i, s_i > 0$ (i =1, 2, ……., n)

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B. Model with Stochastic Budget and Floor Space

Min EC_i ($Q_1,Q_2,\ldots Q_n, s_1,s_2,\ldots s_n$) =

$$\left(\frac{D_i K_i}{Q_i(1-\theta_i)} + h_i(s_i - \mu_i + \frac{1}{2}(Q_i(1-\theta_i)+\theta_i)) + \frac{D_i \pi_i \bar{b}_i(s)}{Q_i(1-\theta_i)} + h_i'\theta_i(Q_i - 1) + \frac{D_i v_i}{1-\theta_i}\right)$$

…..(2.2)

subject to the constraints

$$\sum_{i=1}^{n} \hat{p}_i Q_i \le \hat{B}$$

$$\sum_{i=1}^{n} \hat{f}_i Q_i \le \hat{F}$$

$Q_i, s_i > 0$ (i =1, 2, ……., n)

[Here '^' indicates the randomization of the parameters]

3. FEW STOCHASTIC MODELS

3.1 Demand Follows Uniform Distribution

We assume that lead time demand for the period for the i[th] item is a random variable which follows uniform distribution and if the decision maker feels that demand values for item i below a_i or above b_i are highly unlikely and values between a_i and b_i are equally likely, then the probability density function $f_i(x)$ are given by:

$$f_i(x) = \begin{cases} \frac{1}{b_i - a_i} & \text{if } a_i \le x \le b_i \\ 0 & \text{otherwise} \end{cases} \quad \text{for I = 1, 2, …, n.}$$

So, $\bar{b}_i(s_i) = \dfrac{(b_i - s_i)^2}{2(b_i - a_i)}$ for i = 1, 2, …, n ...(3.1)

Where, $\bar{b}_i(s_i)$ are the expected number of shortages per cycle and all these values of $\bar{b}_i(s_i)$ affects the desired models.

3.2 Demand Follows Exponential Distribution

We assume that lead-time demand for the period for the i[th] item is a random variable that follows exponential distribution. Then the probability density function $f_i(x)$ are given by:

$f_i(x) = \lambda_i e^{(-\lambda_i x)}$, x>0 for I = 1, 2, …, n.

= 0 , otherwise

So, $\bar{b}_i(s_i) = \dfrac{e^{-\lambda_i s_i}}{(-\lambda_i)}$ for I = 1, 2, …, n ….(3.2)

Where, $\bar{b}_i(s_i)$ are the expected number of shortages per cycle and all these values of $\bar{b}_i(s_i)$ affects all the desired models.

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4.MATHEMATICAL ANALYSIS

4.1 Fuzzy Non-linear Programming (FNLP) Technique to Solve Multi-Objective Non-Linear Programming Problem (MONLP)

A Multi-Objective Non-Linear Programming (MONLP**)** or Vector Minimization problem (VMP) may be taken in the following form**:**

$$\text{Min} f(x) = (f_1(x), f_2(x), \ldots\ldots, f_k(x))^T$$

Subject to $x \in X = \{x \in R^n : g_j(x) \le or = or \ge b_j \; for j = 1,2,\ldots\ldots, m\}$ ….(4.1)

and $l_i \le x \le u_i (i = 1,2,\ldots., n)$

Zimmermann (1978) showed that fuzzy programming technique can be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used**:**

STEP 1: Solve the MONLP of equation (4.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

STEP 2: From the result of step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \ldots. \\ x^k \end{array}
\begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \ldots\ldots & f_k(x) \end{array} \\
\begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \ldots. & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \ldots. & f_k(x^2) \\ \ldots. & \ldots. & \ldots. & \ldots. \\ f_1(x^k) & f_2(x^k) & \ldots. & f_k^*(x^k) \end{bmatrix} \end{array}$$

Here $x^1, x^2, \ldots\ldots, x^k$ are the ideal solutions of the objective functions $f_1(x), f_2(x), \ldots\ldots, f_k(x)$ respectively.

So $U_r = \max\{f_r(x_1), f_r(x_2), \ldots\ldots f_r(x_k)\}$

and $L_r = \min\{f_r(x_1), f_r(x_2), \ldots\ldots f_r(x_k)\}$

[L_r and U_r be lower and upper bounds of the r^{th} objective functions $f_r(x) \; r = 1,2,\ldots.., k)$]

STEP 3: Using aspiration level of each objective of the MONLP of equation (6.1) may be written as follows:

Find x so as to satisfy

$$f_r(x) \tilde{\le} L_r \quad (r = 1,2,\ldots\ldots., k)$$

$$x \in X$$

Here objective functions of equation (4.1) are considered as fuzzy constraints. These types of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$$\mu_r(f_r(x) = 0 \text{ or } \to 0 \text{ if } f_r(x) \ge U_r$$

$$= \mu_r(f_r(x) \text{ if } L_r \le f_r(x) \le U_r \;\; (r = 1,2,\ldots\ldots., k)$$

$$= 1 \qquad \text{if } f_r(x) \le L_r \qquad \ldots.(4.2)$$

Having elicited the membership functions (as in equation (4.2)) $\mu_r(f_r(x)$ for r = 1, 2, …… , k, introduce a general aggregation function

$$\mu_{\tilde{D}}(x) = G(\mu_1(f_1(x)), \mu_2(f_2(x)), \ldots\ldots\ldots, \mu_k(f_k(x))).$$

So a fuzzy multi-objective decision making problem can be defined as

Max $\mu_{\tilde{D}}(x)$

subject to $x \in X$ ….(4.3)

Here we adopt the fuzzy decision as:

Fuzzy decision based on minimum operator (like Zimmermann's approach (1976). In this case equation (4.3) is known as $FNLP_M$.

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Then the problem of equation (4.3), using the membership function as in equation (4.8), according to min-operator is reduced to:

Max α ….(4.4)

Subject to $\mu_i(f_i(x) \geq \alpha$ for $i = 1,2,........, k)$

$x \in X \quad \alpha \in [0,1]$

STEP 4: Solve the equation (4.4) to get optimal solution.

4.2 Geometric Programming Problem

Geometric Programming (GP) can be considered to be an innovative modus operandi to solve a nonlinear problem in comparison with other nonlinear techniques. It was originally developed to design engineering problems. It has become a very popular technique since its inception in solving nonlinear problems. The advantages of this method is that, this technique provides us with a systematic approach for solving a class of nonlinear optimization problems by finding the optimal value of the objective function and then the optimal values of the design variables are derived, also. This method often reduces a complex nonlinear optimization problem to a set of simultaneous equations and this approach is more amenable to the digital computers.

GP is an optimization problem of the form:

$Min\ g_0(t)$ …(4.5)

subject to

$g_j(t) \leq 1,$

j = 1, 2, ………, m.

$h_k(t) = 1,$ k=1, 2, ……….., p

$t_i > 0,$ i = 1, 2, ………., n

where, $g_j(t)$ (j = 1, 2, ………, m) are posynomial or signomial functions and $h_k(t)$ (k=1, 2, ……….., p) are monomials t_i (i = 1, 2, ………., n) are decision variable vector of n components t_i (i = 1, 2, ………., n).

The problem (4.5) can be written as:

$Min\ g_0(t)$

subject to

$g'_j(t) \leq 1,$ j = 1, 2, ………, m.

t > 0, [since $g_j(t) \leq 1,\ h_k(t) = 1 \Rightarrow g'_j(t) \leq 1$ where $g'_j(t)(= g_j(t)/h_k(t))$ be a posynomial (j=1, 2, ………, m ; k=1, 2, ………, p)].

I. Posynomial Geometric Programming Problem

A. Primal problem

$Min\ g_0(t)$ …(4.6)

subject to

$g_j(t) \leq 1,$ j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} (i=1, 2, ………,n ; k=1, 2, ………, N_j ; j=0, 1, ………,m) are real numbers.

$T=(t_1, t_2, ………., t_n)^T$.

It is a constrained posynomial primal geometric problem (PGP). The number of inequality constraints in the problem (4.6) is m. The number of terms in each posynomial constraint function varies and is denoted by N_j for each j=0, 1, 2, ……, m.

The degree of difficulty (DD) of a GP is defined as (number of terms in a PGP) –(number of variables in PGP)-1.

B. Dual Problem

The dual problem of (4.4) is as follows:

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Max $d(w)=\prod_{j=0}^{m}\prod_{k=1}^{N_j}\left(\frac{c_{jk}w_{j0}}{w_{jk}}\right)^{w_{jk}}$

Subject to

$\sum_{k=1}^{N_0} w_{0k}=1$ (normality condition)

$\sum_{j=0}^{m}\sum_{k=1}^{N_j}\alpha_{jki}w_{jk}=0,$ (i=1, 2, ………, n) (orthogonality condition)

$w_{j0}=\sum_{k=1}^{N_0} w_{jk}\geq 0$, $w_{jk}\geq 0$, (i=1, 2, ………,n ; k=1, 2, ………, N_j), $w_{00}=1.$

There are n+1 independent dual constraint equalities and $N=\sum_{j=1}^{m}N_j$ independent dual variables for each term of primal problem. In this case DD=N-n-1.

II. Signomial Geometric Programming Problem

A. Primal problem

$Min\ g_0(t)$ …(4.7)

subject to

$g_j(t)\leq\delta_j$, j = 1, 2, ………, m.

t_i > 0, (i =1, 2, …………..,n)

where $g_j(t)=\sum_{k=1}^{N_j}\delta_{jk}c_{jk}\prod_{i=1}^{n}t_i^{\alpha_{jki}}$

here, c_{jk} > 0 and α_{jki} $\delta_j=\pm 1$ (j = 2, …..,m)

$\delta_{jk}=\pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

T=(t_1, t_2, ………., t_n)^T.

B. Dual Problem

The dual problem of (4.7) is as follows:

Max $d(w)=\delta_0(\prod_{j=0}^{m}\prod_{k=1}^{N_j}\left(\frac{c_{jk}w_{j0}}{w_{jk}}\right)^{\alpha_{jk}w_{jk}})^{\delta_0}$ ….(4.8)

Subject to

$\sum_{k=1}^{N_0}\delta_{0k}w_{0k}=\delta_0$ (normality condition)

$\sum_{j=0}^{m}\sum_{k=1}^{N_j}\delta_{jk}\alpha_{jki}w_{jk}=0,$ (i=1, 2, ………, n) (orthogonality condition)

$\delta_j=\pm 1$ (j = 2, …..,m) $\delta_0=+1,-1.$

$\delta_{jk}=\pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

$w_{j0}=\delta_j\sum_{k=1}^{N_0}\delta_{jk}w_{jk}\geq 0$, $w_{jk}\geq 0$, (j=1, 2, ………,m ; k=1, 2, ………, N_j), $w_{00}=1.$

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4. FUNCTIONAL SUBSTITUTION

When a non-linear programming problem (NLP) is of the following form:

$$Min\, y(x) = f(x) + (q(x))^n h(x) \qquad x > 0,\ n > 0.$$

Where, $f(x)$, $q(x)$ and $h(x)$ are single or multi-term functionals of posynomial or signomial form. This generalized formulation is not directly solvable using geometric programming; however, under a simple transformation it can be changed into standard geometric programming form. Let $P = q(x)$ and replace the above problem with the following one:

$$Min\, \bar{y}(x) = f(x) + P^n h(x)$$

subject to

$$P^{-1}(q(x)) \le 1$$

$$x, P > 0.$$

The rationale used in constructing the equivalent problem with an inequality constraint is based on the following logic. Since $y(x)$ is to be minimized, if $q(x)$ is replaced by P, then it is correct to say that $P \ge q(x)$, realizing that in the minimization process P will remain as small as possible. Hence $P = q(x)$ at optimality. Note that $h(x)$ and/or $q(x)$ are permitted to be multiple term expressions and that the optimal (minimizing) solution to $\bar{y}(x)$ is obviously the same as the optimal solution to $y(x)$.

6. FUZZY GEOMETRIC PROGRAMMING PROBLEM

Multi-objective geometric programming (MOGP) is a special type of a class of MONLP problems. Biswal (1992) and Verma (1990) developed a fuzzy geometric programming technique to solve a MOGP problem. Here, we have discussed a fuzzy geometric programming technique based on max-min and max-convex combination operators to solve a MOGP.

To solve the MOGP we use the Zimmerman's technique. The procedure consists of the following steps.

Step 1. Solve the MOGP as a single GP problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions. Repeat the process k times for k different objectives. Let $x^1, x^2, \ldots\ldots\ldots, x^k$ be the ideal solutions for the respective objective functions, where

$x^r = (x_1^r, x_2^r, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, x_n^r)$

Step 2. From the ideal solutions of Step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each solution, the pay-off matrix of size (k x k) can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \cdots \\ x^k \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \cdots\cdots & f_k(x) \end{array} \\ \begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \cdots & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \cdots & f_k(x^2) \\ \cdots & \cdots & \cdots & \cdots \\ f_1(x^k) & f_2(x^k) & \cdots & f_k^*(x^k) \end{bmatrix} \end{array}$$

Step 3. From the Step 2, find the desired goal L_r and worst tolerable value U_r of $f_r(x)$, r = 1, 2, ……., k as follows:

$L_r \le f_r \le U_r$, r = 1, 2, ……., k

Where, $U_r = \max\{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

$L_r = \min\{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

Step 4. Define a fuzzy linear or non-linear membership function $\mu_r[f_r(x)]$ for the r-th objective function $f_r(x)$, r = 1, 2, ……., k

$\mu_r[f_r(x)] = 0$ or $\to 0$ if $f_r(x) \ge U_r$

$= d_r(x)$ if $L_r \le f_r(x) \le U_r$ (r = 1, 2, …… , k)

$= 1$ or $\to 1$ if $f_r(x) \le L_r$

Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

Step 5. At this stage, either a max-min operator or a max-convex combination operator can be used to formulate the corresponding single objective optimization problem.According to Zimmerman (1978) the problem can be solved as:

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$$\mu_D(x^*) = Max(Min(\mu_1(f_1(x)), \mu_2(f_2(x)), ..., \mu_k(f_k(x))))$$

subject to
$g_j(x) \le b_j$, j=1, 2, ….., m, $x > 0$
which is equivalent to the following problem as:
Max α …(6.1)
Subject to
$\alpha \le \mu_r[f_r(x)]$, for r = 1, 2, …….., k
$g_j(x) \le b_j$, j=1, 2, ….., m, $x > 0$
The parameter α is called an aspiration level and represents the compromise among the objective functions. After reducing the problem into a standard form of a PGP problem, it can be solved through a GP technique.

7. MATHEMATICAL ANALYSIS TO MANAGE THE STOCHASTIC CONSTRAINTS

A stochastic non-linear programming problem is considered as:

Min $f_0(X)$
Subject to
$f'_j(X) \le c_j$ (j=1, 2, ………..,m)
$X \ge 0$.

i.e Min $f_0(X)$ ….(7.1)
Subject to
$f_j(X) \le 0$ (j=1, 2, ………..,m)
$X \ge 0$.
Where, $f_j(X) = f'_j(X) - c_j$
Here X is a vector of N random variables $y_1, y_2, ……….,y_n$ and it includes the decision variables $x_1, x_2, ……….,x_n$.
Expanding the objective function $f_0(X)$ about the mean value $\bar{y}_i$ of y_i and neglecting the higher order term:

$$f_0(X) = f_0(\bar{X}) + \sum_{i=1}^{N}\left(\frac{\partial f_0}{\partial y_i}\bigg|\bar{X}\right)(y_i - \bar{y}_i) = \xi(X) \text{ (say)} \quad ….(7.2)$$

If y_i (i=1, 2, ……. ,n) follow normal distribution then so does $\xi(X)$. The mean and variance of $\xi(X)$ are given by:

$$\bar{\xi} = \xi(\bar{X}) \quad ….(7.3)$$

$$\sigma_\xi^{\ 2} = \sum_{i=1}^{N}\left(\frac{\partial f_0}{\partial y_i}\bigg|\bar{X}\right)\sigma_{y_i}^{\ 2} \quad ….(7.4)$$

When some of the parameters of the constraints are random in nature then the constraints will be probabilistic and thus, the constraints can be written as:

$$P(f_j \le 0) \ge r_j \quad \text{(j=1,2 , ………,m)} \quad ….(7.5)$$

Then in the light of the theoretical convention given above, equivalent deterministic constraints are:

$$\bar{f}_j - \phi_j(r_j)\left[\sum_{i=1}^{N}\left(\frac{\partial f_j}{\partial y_i}\bigg|\bar{X}\right)\sigma_{y_i}^{\ 2}\right]^{1/2} \le 0 \quad \text{(j=1,2 , ………,m)} \quad ….(7.6)$$

where, $\phi_j(r_j)$ is the value of the standard normal variate corresponding to the probability r_j.

When some of the parameters of the constraints are fuzzy then the constraints will be imprecise and thus, we are to consider the following theorem:

Theorem

Let $X : \Gamma \to R$ be a normal fuzzy variable with parameters (a, b). For a chosen confidence level α, $\varepsilon \le \alpha \le 1$ if $[Poss(X = x)] \ge \alpha$ then, $x \ \varepsilon\ [X^L_\alpha , X^U_\alpha]$
Where, $XL_\alpha = a - b\sqrt{-\log\alpha}$, $X^U_\alpha = a + b\sqrt{-\log\alpha}$.

Proof: From definition, $\mu_X(x) = \sigma[X^{-1}(x)]$, $x\ \varepsilon$ R.
Now, $[Poss(X = x)] \ge \alpha$

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$\Rightarrow \mu_X(x) \geq \alpha$

when, $X \sim \tilde{N}$ (a, b)

$\mu_X(x) = \exp(-((x-a)/b)^2)$, $-\infty < X < \infty$.

Therefore,

$$-\left(\frac{x-a}{b}\right)^2 \geq \log\alpha$$

$$\Rightarrow -\sqrt{-\log\alpha} \leq \frac{x-a}{b} \leq \sqrt{-\log\alpha}$$

$$\Rightarrow a - b\sqrt{-\log\alpha} \leq x \leq a + b\sqrt{-\log\alpha}$$

If the fuzzy constraint is of the form:

$$\text{Poss}\left[\sum_{i=1}^{n} \tilde{A}_{ij\ 1} x_1 = \tilde{b}_{ij}\right] \geq \alpha_j \quad , \forall j' = 1, 2, \ldots\ldots\ldots, J' \qquad \ldots(7.7)$$

Then, we define J′ normal fuzzy variables as follows:

$$\tilde{Y}_{ij} = \left[\sum_{i=1}^{n} \tilde{A}_{ij\ 1} x_1 - \tilde{b}_{ij}\right], \qquad \forall j' = 1, 2, \ldots\ldots\ldots, J'$$

where, $\tilde{A}_{ij'1}$ *and* $\tilde{b}_{ij'}$ are mutually min-related normal fuzzy variables and

$\tilde{Y}_{ij'} \sim \tilde{N}(m^Y{}_{ij'}, d^Y{}_{ij'})$.

So, the fuzzy constraint (7.7) changes to:

Poss $\left[\tilde{Y}_{ij'} = 0\right] \geq \alpha_{j'}$, $\forall j'$.

Hence, from the above Theorem, we have J′ pairs of equivalent crisp constraints as follows:

$$m^Y{}_{ij'} - d^Y{}_{ij'}\sqrt{-\log\alpha_j{}'} \leq 0,$$

$$m^Y{}_{ij'} + d^Y{}_{ij'}\sqrt{-\log\alpha_j{}'} \geq 0, \ \forall j' \qquad \ldots(7.8)$$

8. NUMERICALS

To solve MOSIM of section 4.1, we use the methods described in the sections 4.2, 5, and 6 and the following data are considered:

Case1. The lead time demand follows uniform distribution and thus $\bar{b}_i(s_i) = \frac{(b_i - s_i)^2}{2(b_i - a_i)}$ for i = 1, 2, …, n , the expected demand short at the end of the cycle takes up the value according to (3.1).

We consider two different set of data as:

D_1=2700; K_1=12; h_1=0.55; θ_1=0.6; μ_1=(a1+b1)/2; v_1=0.03; π_1=1; h'_1=0.25; a_1=20; b_1=70; μ_1=(a1+b1)/2, p_1=3, f_1=2.

D_2=2750; K_2=10; h_2=0.25; θ_2=0.8; μ_2=(a2+b2)/2; v_2 =0.02; π_2=2;

h'_2=0.15; a_2=10; b_2=50; μ_2=(a_2+b_2)/2, p_2=2, f_2=3, B=40000, F=50000.

[All the cost related parameters are measured in "Rs"]

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Table 1: Comparison of Solutions by GP and FGP in Deterministic Environment Under Uniform Lead Time Demand

Method	Q_1	Q_2	s_1	s_2	EC_1	EC_2
GP	422	1064	72	48	521.34	589.65
FGP	437	1051	68	50	516.23	580.54

Case2. The lead time demand follows exponential distribution and thus $\bar{b}_i(s_i) = \frac{e^{-\lambda_i s_i}}{(-\lambda_i)}$ for I = 1, 2, …, n , the expected demand short at the end of the cycle takes up the value according to (3.2).

We consider two different set of data as:

D_1=2700; K_1=8; h_1=1; θ_1=0.4; $\mu_1=1/\lambda_1$; v_1 =0.03; π_1=1; h'_1=0.25; λ_1=1, p_1=3, f_1=2.

D_2=2750; K_2=10; h_2=1;θ_2=0.7; v_2 =0.02; π_2=1.1; h'_2=0.15; $\mu_2=1/\lambda_2$; λ_2=1.1, p_2=2, f_2=3, B=40000, F=50000.

[All the cost related parameters are measured in "Rs"]

Table 2: Comparison of Solutions by GP and FGP in Deterministic Environment Under Exponential Lead Time Demand

Method	Q_1	Q_2	s_1	s_2	EC_1	EC_2
GP	340	945	1.9	2.3	387.33	440.65
FGP	310	939	3.5	2.7	379.65	430.71

To solve MOSIM of section 4.1 with uniform lead-time Demand $\bar{b}_i(s_i) = \frac{(b_i - s_i)^2}{2(b_i - a_i)}$ for i = 1, 2, …, n , using equation (3.1), we use the methods described in the sections 4.2, 5, 6 and 7 and the following data are considered:

D_1=2700; K_1=12; h_1=0.55; θ_1=0.6; μ_1=(a1+b1)/2; v_1=0.03; π_1=1; h'_1=0.25; a_1=20; b_1=70; μ_1=(a1+b1)/2, $\hat{f}_1$ =(2,0.02), $\hat{p}_1$=(3, 0.03).

D_2=2750; K_2=10; h_2=0.25; θ_2=0.8; μ_2=(a2+b2)/2; v_2 =0.02; π_2=2;h'_2=0.15; a_2=10; b_2=50; $\mu_2=(a_2+b_2)/2$, $\hat{p}_2$ =(2, 0.02) $\hat{f}_2$=(3, 0.03) $\hat{B}$ =(40000, 40) $\hat{F}$=(50000,50).

[All the cost related parameters are measured in "Rs"]

Table 3: Comparison of Solutions by GP and FGP in Stochastic Environment Under Uniform Lead Time Demand

Method	Q_1	Q_2	s_1	s_2	EC_1	EC_2
GP	416	1040	67	46	534.22	572.43
FGP	430	1128	62	50	526.29	562.98

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Again, to solve the model of section 4.1 with exponential lead-time demand $\bar{b}_i(s_i) = \frac{e^{-\lambda_i s_i}}{(-\lambda_i)}$ for I = 1, 2, …, n using equation (3.2), we use the methods described in the sections 4.2, 5, 6 and 7 and consider the following data:

D_1=2700; K_1=8; h_1=1; θ_1=0.4; $\mu_1=1/\lambda_1$; v_1 =0.03; π_1=1; h'_1=0.25; $\hat{f}_1$ =(2,0.02), $\hat{p}_1$=(3, 0.03)

D_2=2750; K_2=10; h_2=1;θ_2=0.7; v_2 =0.02; π_2=1.1; h'_2=0.15; $\mu_2=1/\lambda_2$; λ_2=1.1, $\hat{p}_2$ =(2, 0.02) $\hat{f}_2$=(3, 0.03) $\hat{B}$ =(40000, 40) $\hat{F}$=(50000,50).

[All the cost related parameters are measured in"Rs"]

Table 4: Comparison of Solutions by GP and FGP in Stochastic Environment Under Exponential Lead Time Demand

Method	Q_1	Q_2	s_1	s_2	EC_1	EC_2
GP	321	930	1.9	3.2	380.65	430.43
FGP	332	941	3.2	2.3	375.66	421.78

9. CONCLUSION AND FUTURE SCOPE

From the numerical illustrations it is concluded that if we consider the values of EC_1 and EC_2 of all the 4 cases of Table 1 to Table 4, in case of Fuzzy Geometric Programming (FGP) we obtain more minimized values in comparison to Geometric Programming (GP). So, it is observed that FGP technique is better than GP technique for the solution of a Probabilistic Inventory Model.

The model can be analyzed also for lead time demand following Normal Distribution. Besides Stochastic environment, we can illustrate the model in case of Fuzzy environment also.

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