

Application of Deterministic and Stochastic Approaches in Determining the Uncertainty of Claim Reserves.

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Abstract: Accurate estimation of claims is fundamental in the insurance industry for maintaining financial stability and ensuring effective risk management. Traditionally, deterministic approaches, including the chain ladder and Bornhuetter-Ferguson models, have been used to estimate reserves, considering their ease of implementation. However, these models fail to capture the inherent uncertainty associated with unpredictable future variations since they provide point estimates, and hence may result in improper reserve allocation (over-reserving and under-reserving). In contrast, the proposed stochastic model, specifically the bootstrapping technique, introduces a probabilistic framework to quantify reserve variability and provide a distribution of possible outcomes. The goal of this study is to evaluate the effectiveness of the stochastic model as compared to deterministic approaches in quantifying uncertainty, a subject that is largely underexplored in the Kenyan market. In particular, modeling is done for the Incurred but not Reported (IBNR) reserve using real data obtained from a local and already established general insurance company in Kenya (CIC Insurance). Both the deterministic and stochastic approaches are applied on the data, and the model performance is assessed based on accuracy in reserve prediction, mean square errors, confidence intervals, and volatility. The findings demonstrate the advantages of integrating stochastic models in the claim reserving process since they provide a detailed view of uncertainty. The insights support actuarial decision-making and enhance assessments of capital adequacy, hence protecting insurance companies against solvency risks. The study highlights the necessity of integrating stochastic approaches into reserving procedures to enhance robustness of actuarial valuation practices.

Keywords: Uncertainty, Claim Reserve, stochastic, Deterministic.

I. INTRODUCTION

Insurance is an agreement between the policyholder and an insurer, where a series of payments are made (premiums) in order for the individuals to be indemnified against the occurrence of a given event that might result in a loss. Claims are major contributors to the liabilities of an insurance company. Therefore, the insurers need to handle claims efficiently to ensure they meet the policyholder's obligations while maintaining financial stability. Claims reserves represent fund values made available to cover liabilities and expenses related to claim events that have already occurred (past events), including both the Incurred but not reported (IBNR) and the reported claims. Hindley (2017) defines claim reserves as funds estimated and set aside by insurance companies to cover future claim liabilities. It is important to understand the need to accurately gauge future claim costs. If a reserve is under-estimated, the insurance company will fail to fulfil its obligation to the policyholders which implies that the insurer will be exposed to insolvency. If a reserve is over-estimated, it suggests that the company's financial health is at risk since the leverage ratios are affected. Therefore, insurance company needs to be in a position to pay its claims failure to which, the reputation of the insurance company might be ruined.

Traditionally, deterministic models, including the Chain Ladder and Born Huetter-Fergusson approaches, have been widely used to estimate claim reserves since they are easy to implement and interpret. The models generate a point reserve estimates, based on an historical pattern of claim development and fixed assumptions. With this limitation, the models fail to communicate explicitly the uncertainty in the extrapolated reserves. As a result, there is a possibility for over or under-reserving. The modern regulatory frameworks have called the need to go beyond deterministic models to cub the insolvency risks by introducing a probabilistic factor in the modelling process, which in this case the bootstrapping technique is considered a robust alternative. Such stochastic approach enables actuarial analysts to quantify uncertainty risks more explicitly. The research's main objective is to investigate how effective the stochastic models can be compared to deterministic approaches in relation to quantification of uncertainty of claim reserves. The study applied both the deterministic and stochastic approaches to the dataset of cumulative claims from CIC insurance company and metrics such as mean square error, confidence intervals, coefficient of variations, and volatility were evaluated.

The study provides a practical implication of the need to integrate stochastic models a non-conditional aspect rather than a supporting mechanism in reserve projection.

II. LITERATURE REVIEW

Obtaining the metrics to quantify claims reserves in general insurance is a key area of study in the insurance sector. According to the 2014 Insurance Regulation of Kenya report, general insurance contributes about 75% of total claims in the insurance sector. Manno and Pereira (2009) classified reserves into two main groups, i.e., reported but not settled (RBNS) and incurred but not reported (IBNR) claim reserves. The authors ascertained that such reserves are estimates and given the reported and paid claims, the incurred and not reported claims can be projected.

Weindorfer (2012) outlined the significant stages to be followed when applying the basic chain ladder model:

- Grouping of claim data and fitting it into run-off/delay triangles.
- Utilizing development factors to calculate development patterns
- Estimating the final claim amounts using the above results.
- Estimating the claim reserve.

Taylor & McGuire (2016) outline that the basic chain ladder model was introduced into the insurance sector for the first time in 1953 when it was by an upcoming insurance company to estimate its reserve. Bourhuetter and Fergusson (1972) advanced the model by factoring in the expected loss indicators, which was to refine the obtained estimates before obtaining the final claim estimates. Mack (1993) questioned the need for statistical quality of claim reserve estimates, an idea that was made possible due to advanced computer technology. Mack introduced a calculation of the standard error on reserve estimates obtained from the commonly used deterministic model, the chain ladder method; however, the model was considered to have no probability distribution. England and Verrall (2002) provide detailed information on how the stochastic models can be used in general insurance claim reserving and give a list of applicable approaches. England and Verrall (2006) advanced the (2002) study and demonstrated the procedures and effectiveness of obtaining claim reserve estimate distributions using the Bayesian and bootstrapping techniques. However, the study concludes that the approaches are essential in generating predictive distributions. In this project, we shall examine how stochastic models, specifically the bootstrapping technique, can be used alongside the deterministic models in order to quantify the uncertainty of claim reserves and generate accurate point estimates.

III. METHODOLOGY

We acknowledge Bruno Dominguez Ramos de Carvalho for the concepts used in this methodology. In their study, the authors focused on measuring the uncertainty of claims reserves by using bootstrapping techniques to the deterministic approaches on data obtained from third-party motor insurance in the Brazilian insurance industry (Carvalho and Carvalho, 2019).

Data used in this project was obtained from quarterly reports of the Insurance Regulatory Authority (IRA), converted into annual accumulations and then analysed using Python software. Data included reported and pain cumulative amounts from the first quarter of 2016 to the last quarter of 2024. Modelling of the data was conducted for both the deterministic and stochastic approaches.

A. A statistical model for run-off triangles

Run-off triangles can generally be expressed as:

Accident Year	Development Year					
	0	1	...	j	...	n
0	C _{0,0}	C _{0,1}	...	C _{0,j}	...	C _{0,n}
1	C _{1,0}	C _{1,1}	...	C _{1,j}	...	C _{1,n-1}
.
.
i	C _{i,0}	C _{i,1}	...	C _{i,j}	...	C _{i,n-i}

Every C_{ij}, in the run-off triangle above represent the incremental claims and is expressed as:

$$C_{\{i,j\}} = f_j \cdot R_i \cdot K_{\{i+j\}} + \varepsilon_{\{i,j\}}$$

f_j is the development factor for year j . Each f_j is independent of the origin year i .
 R_i is a varying parameter influenced by origin year, i representing the exposure.
 K_{i+j} is the parameter that changes based on the calendar year, for instance, inflation.
 e_{ij} is an error term (Schmidt, 2006).

B. The Chain Ladder Algorithm.

Known Future Payments

$C_{1,1}$ $C_{1,2}$ $C_{1,3}$ $C_{1,4}$

$C_{2,1}$ $C_{2,2}$ $C_{2,3}$ $C_{2,4}$

$C_{3,1}$ $C_{3,2}$ $C_{3,3}$ $C_{3,4}$

$C_{4,1}$ $C_{4,2}$ $C_{4,3}$ $C_{4,4}$

The red part present the future payments.

The algorithm is based on the idea that all accident years have the Similar Behaviors (Schiegl, 2015).

$$C_{i,j+1} = C_{i,j} \cdot f_k \quad (1)$$

The development factor f_k is given by,

$$f_k = \frac{\sum_{i=1}^{I-j-1} C_{i,j+1}}{\sum_{i=1}^{I-j-1} C_{i,j}} \quad (2)$$

With Chain Ladder factors, the ultimate claim $C_{i,J}$ for $i + J > I$ at time $t = I$ can be predicted by;

$$C_{i,j} = C_{i,1-j} \prod_{j=I-i}^{j-1} f_j \quad (3)$$

Therefore, given time $t=I$ for accident year $i>I-J$, the reserve is given by:

$$R_i = C_{i,j} - C_{i,1-i} \quad (4)$$

Given an aggregated amount for all years, the outstanding loss liability reserve is;

$$\sum_{i=I-J+1}^J R_i \quad (5)$$

C. Assumptions of the Basic Chain Ladder Model

1. Given the development factors $f_1, f_2, \dots, f_{n-1} > 0$;

$E(C_{i,k+1} | C_{i,1}, \dots, C_{i,k}) = C_{i,k} f_k, 1$. The formula is an illustration of a markov chain. This means that the expected value of future claim liabilities is based on the recent cumulated claims rather than entire history (Moaait, 2023).

2. $C_{i,1}, \dots, C_{i,n}, \dots, C_{j,1}, \dots, C_{j,n}$ are independent; meaning, claim developments have no relationship with the different occurrence periods.

D. Bornhuetter and Fergusson model

The Bornhuetter-Ferguson model complements the chain ladder approach by incorporating an indicator of the expected loss ratio. The estimate of claim amounts is given as;

$$C_{i,n,BF} = \beta_i \cdot \pi_i \cdot \left(1 - \frac{1}{f_k}\right) \quad (6)$$

Where;

β_i is the expected claim for each occurrence period i .

π_i is the amount of premium earned for each occurrence period i .

f_k is the cumulative development factor.

The IBNR reserve is then obtained as;

$$R = \sum_{i=0}^n C_{i,nBF} - C_{i,n+1-i} \quad (7)$$

E. Mean Square Error

For the mean square errors to be obtained, the following assumptions need to hold:

1. The total number of expected claims for each period in development period t is obtained by multiplying the development factor from time $t-1$ to t , by the claim amount in $t-1$ (Weindorfer, 2012).

$$fk = \frac{\sum_{i=1}^{n-k} Ci,k+1}{\sum_{i=1}^{n-k} Ci,k} \quad (8)$$

2. All claim amounts are independent given different occurrence periods.
3. The variations of development factors in different period intervals decrease as the starting claim amounts in the interval increase

With the above assumptions, the mean square error (standard deviation) of claim reserves is obtained for each period i [mse (Ri)]:

$$Mse(R_i) = mse(C_{i,n}) \quad (9)$$

That is, the average squared error of claim reserves for each period i is equal to the average squared error of cumulative claims on period n .

$$mse(R_i) = C_{i,n}^2 \sum_{k=n+1-i}^{n-1} \frac{\sigma_k^2}{fk} \left(\frac{1}{Ci,k} + \frac{1}{\sum_{j=1}^{n-k} Cj,k} \right) \quad (10)$$

whereby;

$$\sigma_k^2 = \frac{1}{n-k-1} \sum_{i=n}^{n-k} Ci,k \left(\frac{Ci,k+1}{Ci,k} - fk \right)^2, 1 \leq k \leq n-2 \quad (11)$$

σ^2 an unbiased estimation for the total cumulative claim amount.

The average square error for the total reserve value is estimated as;

$$Mse(R) = \sum_{i=2}^n \left\{ (s.e(R_i))^2 + Ci,n \left(\sum_{j=i+1}^n Cj,n \right) \sum_{k=n+1-i}^{n-1} \frac{2\sigma_k^2 / f^{2k}}{\sum_{n-1}^{n-k} Cn,k} \right\} \quad (12)$$

F. A Stochastic Approach for Estimating Claim Reserves

The bootstrapping technique involves re-estimation of cumulative values based on the previous occurrence period in which the development factors in the last diagonal of the reported cumulative claims triangle are maintained for simulations. After simulation, the Pearson residuals are derived from the difference between the initially obtained triangle of cumulative claim data and the current re-estimated triangle using the following equation:

$$r_{ik}^{(p)} = \frac{Cik - mik}{\sqrt{mik}} \quad (13)$$

where m_{ik} represents an estimate of the incremental reported claim value derived from the p -th round of a bootstrap. In the next step, a scale parameter is obtained for the residual so that given scale, new random triangles can now be derived as below;

$$A = \frac{\sum_{i,j,k-i+1} (r_{ik}^{(p)})^2}{\frac{1}{2}n(n+1) - 2n+1} \quad (14)$$

In the final step, the Pearson residuals are refined to correct bias through the below analytical approach:

$$r_{ik}^{(adj)} = \sqrt{\frac{n}{\frac{1}{2}n(n+1) - 2n+1}} \cdot r_{ik}^{(p)} \quad (15)$$

Through the Pearson residual process, various residual triangles are generated. Given the generated residual triangles, new triangles of reported claims are built, including reserve and claim amount estimates. The process is repeated then k times (for this research, $k = 100,000$), and the set of values obtained forms an empirical distribution

IV. DATA ANALYSIS

A. Cumulative Data

The cumulative claims runoff triangle below represents total amount of reported and paid claims from the year 2016 to 2024.

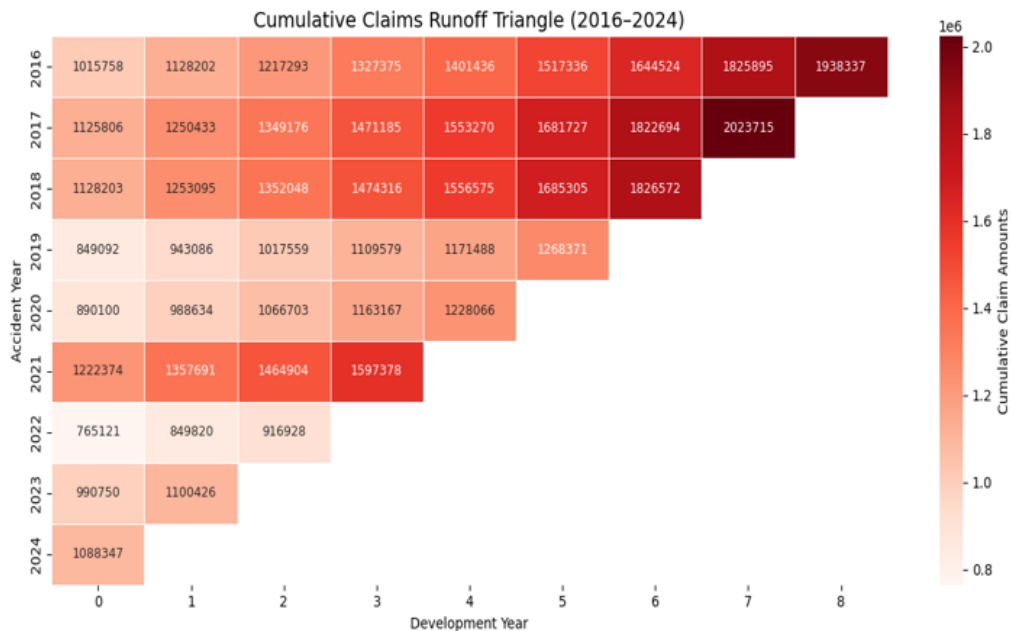


Fig. 1 cumulative claims

B. Chain Ladder IBNR Reserve Estimates

After fitting equation (2) for development factors, equation (3) and (4), the IBNR reserve estimates for the chain-ladder model are obtained as below:

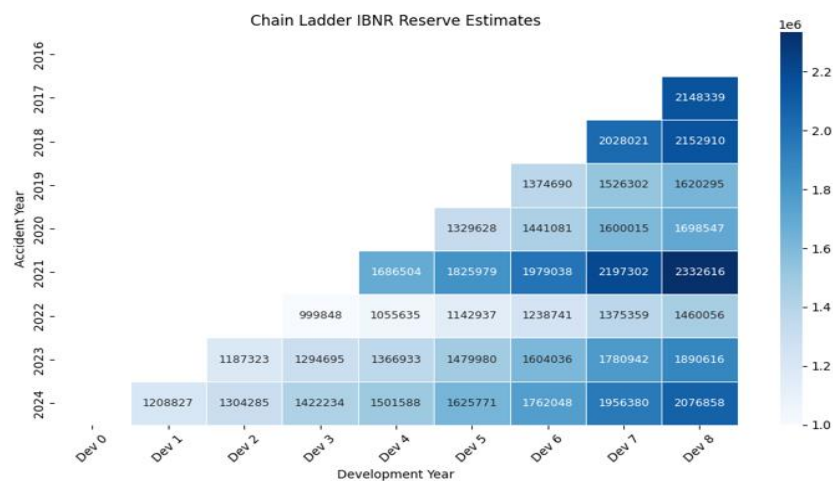


Fig. 2 chain ladder IBNR reserve estimates

From Fig. 2, the total IBNR reserve estimate is KES59,918,91.

C. Bornhuetter-Fergusson IBNR Reserve

Complementing the above chain ladder approach and incorporating the expected loss ratio indicator, and applying equation (6) and (7), the below figure presents the IBNR reserve using Bornhuetter-Fergusson method.

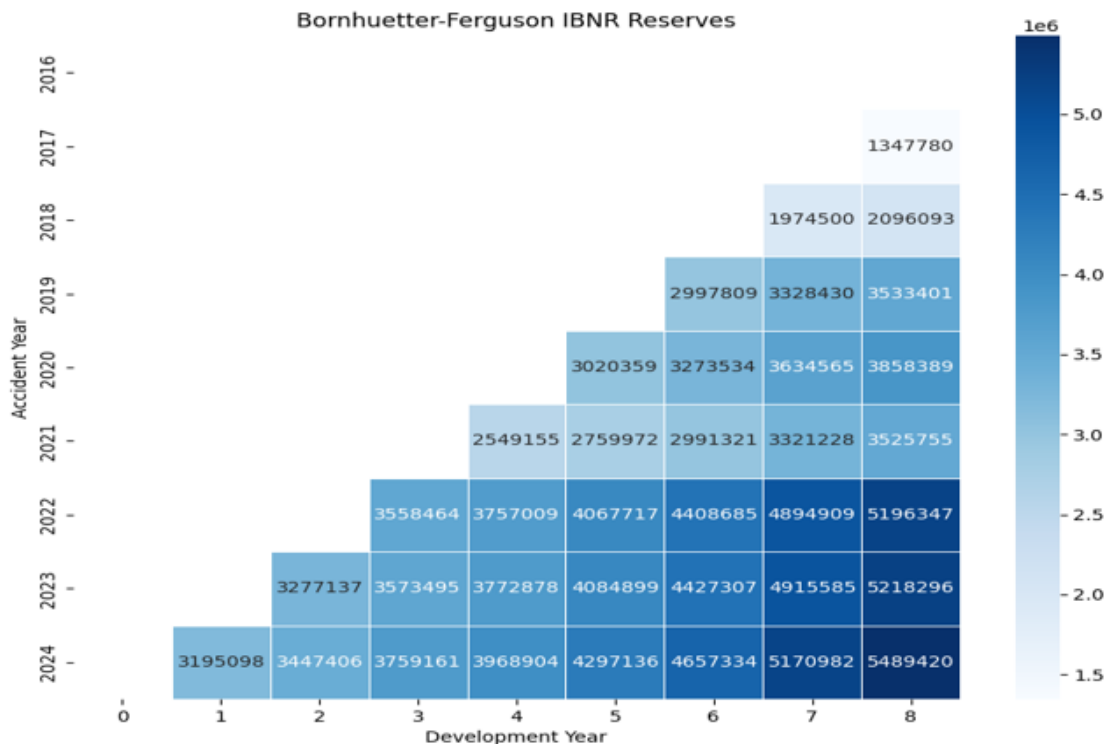


Fig. 3 Bornhuetter-Fergusson IBNR reserve

From Fig. 3 the total IBNR reserve estimate (single point estimate) is KES61,651,259.

D. Mean Square Errors

From equation (11) individual variances for each occurrence period i using the chain ladder and bornhuetter-Fergusson methods were obtained as tabulated below. From equation (9) and (10), the average squared error of claim reserves for each period i from the deterministic approaches were obtained as presented in the table below. Also, from equation (12), the average square error for the total reserve value ($MSE(R)$) were estimated.

Table 1: Mean Square Errors
MSEs for Chain Ladder (CL) and Bornhuetter-Fergusson (BF)

Year	Variance (CL)	MSE(Ri) (CL)	Variance (BF)	MSE(Ri) (BF)
1	0.00	0.00	0.00	0.00
2	401,225.24	0.00	1,883,052	0.00
3	382,629.32	138,221,400,000	298,325	81,219,460,000
4	105,032.84	316,006,600,000	216,463	276,936,000,000
5	69,798.03	625,359,800,000	137,098	719,112,300,000
6	107,178.24	487,040,500,000	50,793	2,442,093,000,000
7	111,026.91	1,423,691,000,000	24,692	3,860,272,000,000
8	-	2,278,226,000,000	-	12,134,900,000,000
MSE(R)		5,268,545,160,560		19,514,533,231,192

Variance illustrates the uncertainty in predicted estimates due to variabilities in historical data. Mean square error informs on the accuracy of the predicted estimates by factoring in bias and consistency of possible over or under-estimation. From the early development years, both models have produced zero values for both the variances and $MSE(R_i)$.

Such results are expected since the claims data from the period are fully developed. In year 3-5, there is a significant diverging pattern in variances and MSEs. The discrepancies are majorly influenced by the model mechanisms. The total MSE (R) values produce a 270% contrast since CL overlies on observed historical data while BF incorporated expected loss values.

E. Bootstrapping Technique

From equation (13) the Pearson residuals were derived from the difference between the initially obtained triangle of cumulative claim data and the current re-estimated values for each p-th round of the bootstrap. Equation (14) was integrated to set the scaling parameter, while from equation (15), the analytical approach was refined to correct bias. The table below presents the IBNR reserve estimate values in percentile confidence intervals resulted from 100,000 iterations.

Table 2: Confidence Intervals

95% Confidence Interval			75 % Confidence Interval			50% Confidence Interval		
Accident year	Lower Bound	Upper Bound	Accident Year	Lower Bound	Upper Bound	Accident Year	Lower Bound	Upper Bound
1	0	0	1	0	0	1	0	0
2	1076789	5341432	2	1732161	4192214	2	2093166	3809014
3	1157967	5399136	3	1820916	4269595	3	2176484	3882685
4	1166271	5410094	4	1840676	4272503	4	2185232	3891503
5	1133946	5346276	5	1813700	4237234	5	2155310	3853482
6	1538245	5756239	6	2212071	4640422	6	2555367	4259035
7	1278609	5551593	7	1966964	4394572	7	2297711	4008963
8	1642551	5852695	8	2314490	4747290	8	2662344	4362943
9	9584440	13807632	9	10262157	12686206	9	10600984	12298377
Total IBNR Reserve	27234786	40101762		29640588	37218075		31119288	35576463

In case the estimates fall below the lower bound value, there will be a risk of under-estimation and the company is at risk of insolvency. If the estimated claim reserve value lies outside the upper limit, there will be a risk of over-estimation.

F. Comparative Analysis

The spread of the histogram (Figure 4) below demonstrates the volatility of the actual IBNR reserves, which underscores the need for stochastic modelling in the modern actuarial practice especially when making informed decisions on reserve margins and capital sufficiency.

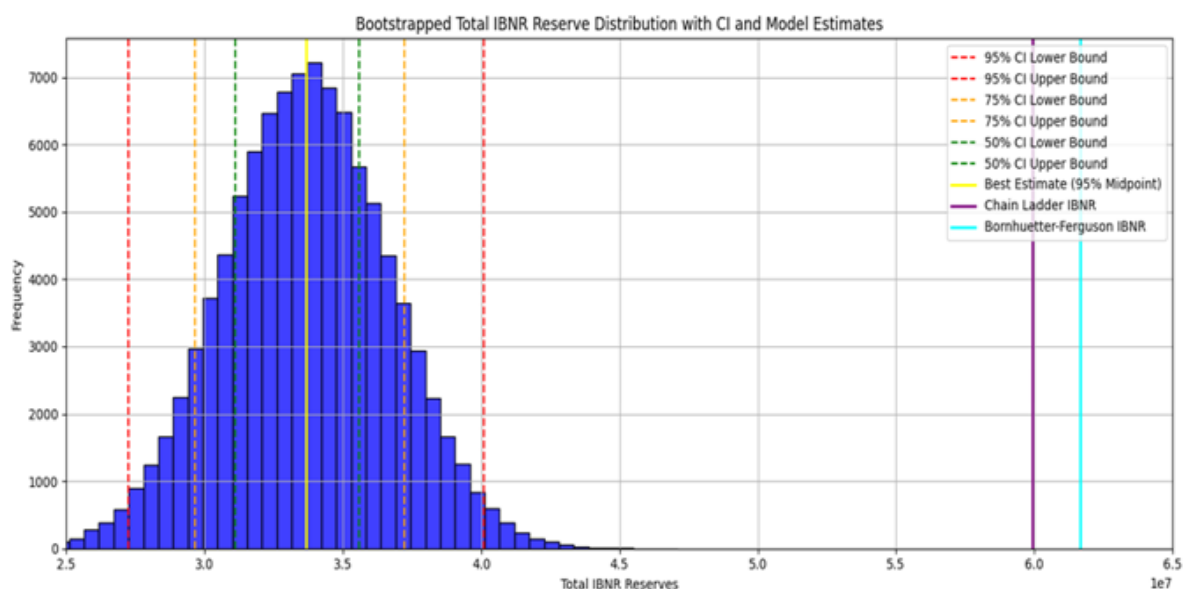


Fig. 4: Bootstrapped total IBNR reserve distribution with CI and model Estimates

The figure 4 presentation highlights the deterministic model reserve estimates falling to the far right of the bootstrapped distribution, notably lying outside the 95% confidence interval. The output implied potential bias toward reserve over-estimation since it exceeded the central mass of distribution.

Table 3: IBNR Reserve Comparison around best estimate

Method	IBNR Reserve Estimate	Difference from CL	% Difference from CL
Chain Ladder (assumed best estimate)	59,918,916	0	0.00%
Bornhuetter-Ferguson	61,651,259	+1,732,343	+2.89%
Bootstrapped (95% Mean)	33,395,347	-26,523,569	-44.27%

From the above table 3, a substantial deviation was observed in the difference between the bootstrapped mean and the best estimate of -26,523,569 which is -44.27 % lower. The bootstrap approach does not solely focus on calculating reserve based on assumptions in place; it involves several simulations that develops alternative scenarios based on variations in historical development factors. As in this case, the bootstrapped mean output highlights that on average, the IBNR reserve could be lower than that reflected by the assumed best estimate. This is a significant illustration that the point estimates are overly conservative and thus fail to reflect uncertainty originating from claim distribution and development. Thus, the outcome highlights the risk of over-reserving.

Table 4 below presents the results of the analysis measures of uncertainty and variability across models.

Table 4: IBNR Reserve standard deviation and coefficient of variation comparison

Method	Total IBNR Reserve	Std. Deviation	Coefficient of Variation
Chain Ladder (Best Estimate)	59,918,916	2,295,328	0.0383
Bornhuetter-Ferguson	61,651,259	4,416,421	0.0716
Bootstrapped (95% Mean)	33,395,347	3,283,469	0.0983

The chain ladder method having a small coefficient of variation and a low standard deviation. While the results may imply low dispersion, it does not guarantee great accuracy, rather a deficiency of not modelling variability explicitly. The Bornhuetter-fergusson shows a high standard deviation f all but a moderate coefficient of variation of 7.16%. The interpreted here indicates more uncertainty, considering the model combines the expected loss assumptions with historical claim development patterns. Overall, the bootstrapped method highlights a medium standard deviation, and highest coefficient variation of 9.38%, which reflects how the approach captures uncertainty and variability explicitly, which is more realistic than the distribution masked by the deterministic models.

V. CONCLUSION

Accurate claim reserve estimation has become a concern in the modern insurance practices with a major focus on quantification of uncertainty. The research study assessed the effectiveness of stochastic approaches compared to deterministic models in capturing variations associated with claim development. Based on the results generated, there is a significant indication of the need to adapt the simulation process generated through bootstrapping given that it factors in various claim development situations that could have taken place given that the claims already occurred. Typically, the deterministic approaches provide a single point estimate, which is a baseline projection, and this limits the insight of how the actual claim reserves can vary, how the risk margins are calculated and suppresses the analysis of extreme scenarios, hence possible risk of over and under-estimation. The results reinforce the growing body of literature by advocating for stochastic approach consideration during claim reserve projections. However, the transition has limitations considering that research solely depend on the past data from the reported and paid liability claims to calculate the incurred but not reported claim reserves. This means that the research is limited to changes in regulatory frameworks, the underwriting policies, inflation shocks, and product development. For this case, it is important that with each passing period, a consistency test is conducted on the existing data based on the available new information to ensure that the models adhere to the real-world expectation of the company's operations. Identifying and understanding the weaknesses of the applied models in determining uncertainty of claim reserves is a foundation for future areas of study and innovations significant to improve reserve modelling.

Future research could consider integrating algorithms that are effective in detecting a structural break in case a systemic change occurs when applying reserving methods. Actuaries could respond appropriately to change in claim behaviour, which will potentially enhance reserve accuracy.

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