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Beyond the Bell Curve: Characteristic Function-Based Value at Risk (VaR) under Stochastic Volatility in Emerging Markets

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Abstract: In this paper, we examined the risk character of the NSEASI index across 10 years (January 1, 2013 - August 31, 2023), consisting of around 2,590 valid trading days following intensive cleaning and outliers adjustment of the data. A daily log return was calculated and shown as a high-risk, low-reward market, with average log returns of 0.0018 and an 11.73% daily volatility. It had extremely high kurtosis (328.199) and almost zero skewness (0.009), implying that the distribution of returns was very skewed to extremes and was not skewed. The characteristic function-based Value at Risk (VaR) model was applied in a stochastic volatility system to rectify the flaw of traditional risk models in the face of this heavy-tailed behaviour. Realistic stochastic dynamics of volatility of returns were obtained using parameter estimation using the method of moments. Comparative analysis using Delta, Delta-Gamma, and Monte-Carlo simulation techniques revealed that the fat-tailed behaviour of the return distribution was better captured when using the CF-based and Monte-Carlo-based approaches. The estimates of VaR at the 5% and 1% confidence levels based on CF (2.80 and 5.10) were significantly higher than those of the Delta and the Delta-Gamma method, which underestimated tail risk. It provides formal backtesting via the Kupiec and Christoffersen tests. It performs a sensitivity analysis and discusses policy implications in the context of financial regulation and corresponding portfolio risk management. We would conclude that CF-based VaR is a more practical and theoretically-grounded alternative to more common methods, in non-Gaussian settings that characterize emerging markets; nevertheless, our findings demonstrate the shortcomings of standard Gaussian-based models in turbulent emerging markets like Kenya. This article recommends the use of advanced stochastic methods in the field of financial risk management and regulation. Future research opportunities include introducing the dynamics of jump-diffusion processes, modeling interdependencies at the constituent level, and improving the dynamic portfolio risk estimation.

Keywords: Value at Risk (VaR), Stochastic Volatility, Characteristic Function, Emerging Markets, NSEASI Index.

I. INTRODUCTION

Modern risk management relies heavily on determining potential losses in financial portfolios, and Value at Risk (VaR) has become a standard measure (Halkos & Tsirivis, 2019). VaR lets you estimate a portfolio's highest possible loss during a specified holding period at 95% or 99% confidence. It is part of setting regulatory capital standards and internal risk measures (Sarykalin et al., 2008). Being able to rely on VaR depends on having the correct assumptions about how asset prices are distributed. The rise of the emerging markets into the world of finance, making it more globalized than ever before, has necessitated the establishment of strong risk management models that consider structural volatility, fat tails, and a market that is not symmetrical in its responses. These properties are not reflected well in the traditional Value-at-Risk (VaR) models, especially in the non-Gaussian distributions of returns. The paper proposes a new characteristic function-based method of estimating VaR in the case of stochastic volatility and compares it to standard and advanced VaR approaches. However, these assumptions may not be adhered to very rigidly since they may be violated with the Nairobi Securities Exchange All Share Index 25 (NSEASI). Empirical evidence revealed that the returns in alternative markets possess non-Gaussian characteristics and, therefore, can not be employed by the old VaR models, which assume the normality of returns (Pagliaro, 2025). It is also hard to test the risks with such guidelines since there is little history and inefficiency.

Earlier, risk managers usually applied parametric methods such as the delta and delta-gamma approximations, which assume linear or quadratic responses of the portfolio to shifts in asset prices (Sulistianingsih et al., 2019). Because they are fast to run, these techniques suppose that returns are usually distributed together, so they miss some risk factors and may underestimate risk, mainly in cases where options are held. At the opposite end is the Monte Carlo simulation, which gives more precise risk estimates by modelling the entire distribution of returns but usually takes much more computing power and adds to the complexity of the model (Glasserman et al., 2019).

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This paper attempts to deal with these deficiencies by consulting the approach suggested by El-Jahel et al. (1999), which is initially designed for a derivative portfolio. We borrow this feature-based formulation of the stochastic volatility framework to explain the returns of the NSEASI index. Our method accepts portfolio returns' quadratic (nonlinear) character (Ghezzi et al., 2024). It implements more practical suppositions regarding the stochastic behaviour of the volatilities of assets, wherein such features are especially relevant in association with the frontier and emerging markets (Webb, 2024). As per Cui et al. (2013), the main point would be that despite the exact distribution of the returns of the portfolio might be hard to calculate analytically, especially in the case where returns are developed as nonlinear of the stochastic processes involved, their moments (mean, variance (of return), skewness, kurtosis, and so on) could still be obtained so far the portfolio can be approximated through the quadratic form. These arise as the derivative of the characteristic function of the underlying stochastic volatility process discussed as a vector. Having acquired this, the distribution of returns can be approximated with moment-matching estimators using flexible parametric families, including Pearson or Johnson distributions, thus making it possible to calculate VaR more precisely (Brignone et al., 2021). The suitability of such a practice can be accentuated by the fact that it can help to cope with the simplistic linear procedures and complex simulation models. It can be assumed that it can provide better accuracy in constructs where the exposures by non-linearity, stochastic volatility, and non-normal distributions of returns without excessive computational expense (In addition to that we are also carrying out the process of statistical backtesting and sensitivity analysis to ensure the strength of each model and also offers realistic suggestions to regulators and institutional investors as well.

II. VAR AND INDEX RETURNS

2.1 Limitations of Traditional VaR Methods

Amin et al. (2018) assumed that VaR measures the most significant estimated loss in a given financial position or portfolio over a definite period, often equated to one trading day, at a selected confidence level. As a case in point, the 1-day VaR at 5 percent indicates that 5 percent of the number of trading days the portfolio is likely to lose money above the VaR level. VaR is therefore a probabilistic measure of the most pessimistic losses in normal market circumstances. There are two significant VaR estimation methods: parametric and nonparametric (Mentel, 2013). Parametric models attempt to parameterize returns that are usually assumed to be normal, as is the case with the variance-covariance (or Delta-normal) approach. VaR depends on the average and volatility of returns that may be collected from actual historical information (Prakash et al., 2021). Since computing this figure is simple, it overlooks skewness and kurtosis, as it presumes that volatility is always the same in emerging markets, which is not true.

According to Simardone and Racine (2021), the historical Simulation is nonparametric because it uses direct sampling from available data to calculate VaR. You should try these algorithms with strange data; they are best when there is a lot of historical data for them to work with. Another chance is that they do not show new regimes or key developments since records are limited to historical data.

Because of flaws in the market, lack of liquidity, unclear politics, and sudden outside shocks, the returns on the NSEASI are often non-Gaussian (Su et al., 2014). They cause both VaR formulas to perform less reliably because they result in substantial volatility and larger unexpected losses.

For this reason, primary reliance on these models might hide significant hazards, which means companies could end up with insufficient coverage and limited management choices—utilising stochastic volatility results in more realistic return distributions, not limited to Gaussians (Hross et al., 2014). The characteristic function method and a stochastic volatility model (for instance, the square-root process) can be reliable alternatives. This method permits semi-automated calculation of VaR, keeping in mind the special statistical qualities of emerging market returns (Rockafellar & Uryasev, 2013).

2.2. Models

2.2.1. Delta Method

The delta method is a parametric approach that approximates portfolio returns using a first-order (linear) Taylor expansion (Sulistianingsih et al., 2019). It counts on high volatility and returns from a normal distribution, which helps make the calculations fast and easy. However, these assumptions are commonly untrue in emerging markets since the distributions of returns often display skewness, kurtosis, and changing volatility. For Delta, the NSEASI is a single investment that can be assigned a value: St. The asset dynamics are modelled using a stochastic differential equation (SDE) that incorporates stochastic volatility:

$$\mathrm{dSt} = \mu S_t d_t + \sigma S_t \sqrt{v_t}\, \mathrm{d}W_t^{(s)} \qquad (1)$$

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where
μ is the drift,
σ is the volatility scale,
v_t is defined to be the stochastic volatility and
$dW_t^{(s)}$ Given the Brownian motion.
The volatility follows:

$$dv_t = [\kappa(\theta - v_t) - \xi v_t]dt + \eta\sqrt{v_t}dW_t^{(v)} \tag{2}$$

With correlation $dW^{(s)}dW^{(v)} = \rho dt$. The return is approximated as:

$$\frac{\Delta S_t}{S_t} = \mu\Delta t + \sigma\sqrt{v_t}\Delta W_t^{(s)} \tag{3}$$

Given the above considerations, we suppose the Gaussian returns have variance.

$$\sigma_\delta^{(2)} = \sigma^2 v_t \Delta t$$

Returns are viewed as normally distributed, having a mean of $\mu\Delta t$ and a variance. $\sigma_\delta^{(2)}$. This delta-normal approach is a standard way to calculate Value at Risk (VaR). This assumption misunderstands certain financial features in the NSEASI, leading to less attention to tail risk, especially in emerging markets (Prakash et al., 2021). For this reason, the delta method cannot fully represent the extreme losses that sometimes happen, so stronger models that focus on characteristic functions and changing volatility are needed.

2.2.2. Delta-Gamma Method

The approach uses second-order sensitivities (gamma) to calculate the VaR (Jabeen & Ilie, 2024). Even though it is more precise than delta-only models, it still believes in Gaussianity and might not properly indicate what makes the market volatile. The delta-gamma method accounts for changes brought by higher-order terms:

$$\frac{\Delta S_t}{S_t} = \frac{\partial S}{\partial S}\frac{\Delta S_t}{S_t} + \frac{1}{2}\frac{\partial^2 S}{\partial S^2}\left(\frac{\Delta S_t}{S_t}\right)^2, \tag{4}$$

With variance

$$\sigma_\delta^{(2)} = \left(\frac{\partial S}{\partial S}\frac{1}{S}\sigma^2 v_t + \frac{1}{2S}\frac{\partial^2 S}{\partial S^2}\sigma^2 v_t^2\right)\Delta t. \tag{5}$$

III. CHARACTERISTIC FUNCTION APPROACH

Basic methods such as the Delta and Delta-Gamma approaches usually do not work well with derivatives with uneven payouts because they cannot deal with higher-order effects, non-random distributions, and unpredictable volatility (Dupret et al., 2022). Because of these limitations, a more powerful technique using characteristic functions (CF) of log prices is introduced. We have a Fourier-based CF model built on Fourier techniques. This approach has been used to do analytical computations of return distributions employing the characteristic function of a stochastic process instead of complete probability density estimation.

The main advantages of the characteristic function-based VaR model are as follows: The ability to capture skewness and kurtosis soundly in the distribution of returns, thus the characteristic function-based VaR model is most competent for markets that are not Gaussian. It is also computationally tractable, and solutions may be obtained analytically or semi-analytically, avoiding exhaustive simulations. Additionally, unlike the arctic environment, it has been proven to be very versatile, as its natural extensions to more realistic financial models using jumps, stochastic volatility, or Levy processes make it more applicable to modelling real-world market effects. The idea of normality is a limitation in VaR modelling, and to solve this, several non-Gaussian techniques have come up. EVT is tuned to the tail events and appropriately applied in the stress test of extreme losses; however, this theory ignores the entire distribution of returns. The advantage of Historical Simulation is that it does not involve parametric assumptions and is simple, but large volumes of data are needed, and it is resistant to outliers.

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Models of the GARCH family apply to modelling volatility clustering, although, in general, they also assume conditional normality and thus are not helpful in tail risk modelling either. Conversely, the CF-based method is bespoke and nonparametric and can accommodate both skewness and kurtosis. It can also model complex processes such as stochastic volatility and jumps. The primary disadvantage is that it requires complicated implementation in mathematics and might not be used by most people without technical expertise.

As such, main moments can be calculated based on the characteristic equation as they usually include the mean, variance, skewness, and kurtosis since they exhibit the full distribution of the random Xt (Neuberger, 2012). Employing a method such as the Edgeworth expansion, we can estimate the Value at Risk (VaR), where we will assume we already have an estimated distribution (Dupret et al., 2022).

$$VaR_\alpha = \mu + z_\alpha\sigma + \frac{(z_\alpha^2 - 1)y_1\sigma}{6} + \frac{(z_\alpha^3 - 3z_\alpha)y_2\sigma^2}{24}$$

Where:

- M is the mean of log returns
- σ is the standard deviation
- γ1 is skewness
- γ2 is excess kurtosis
- zα is the standard normal quantile for confidence level α

It uses the structure of the multivariate Heston model, and each asset price has a stochastic volatility process St. The model also allows the variance of one asset to fluctuate over time due to correlated Brownian motions to capture the relationship among the assets instead of assuming zero correlation (Glasserman & Kim). If portfolios possess multiple assets to reduce risk, then the risk could be measured by considering the stochastic volatility and the correlated shocks (Suleymanov et al., 2024). Expected return of NSEASI is modelled based on its relationship with log price and volatility. VaR is acquired by obtaining the moments with the help of a Pearson family characteristic function.

3.1. Stochastic Model

In this study, we utilise the Heston model framework:

$$dSt = \mu S_t d_t + \sigma S_t \sqrt{v_t}\, dW_t^{(s)} \tag{6}$$

We suppose that the NSEASI value S_t follows equation (6).
The volatility is given as

$$dv_t = [\kappa(\theta - v_t) - \xi v_t]\mathrm{d}t + \eta\sqrt{v_t}\mathrm{d}W_t^{(v)} \tag{7}$$

With correlation $dW^{(s)}dW^{(v)} = \rho dt$.
We define $Xt = \log(St)$. By Itô's lemma

$$dx_t = \left[\mu - \frac{1}{2}\sigma^2 v_t\right]dt + \sigma\sqrt{v_t}\, dW_t^{(s)} \tag{8}$$

Where;
Where S_t is the asset price,v_t is variance, κ. The speed of mean reversion, θ the long-term variance, and σ. The volatility of variance. This model can reflect the mean-reverting stochastic volatility and observed characteristics of emerging markets, such as volatility clustering, leptokurtosis, and negative skew characteristics, which are poorly addressed by the usual VaR approaches.

3.2. Characteristic Function

Here, we derive the joint conditional characteristic function of x_T and v_T as shown:

$$\mathrm{f}(\mathrm{x}, \mathrm{v}, \mathrm{t}; \phi) = \mathrm{E}_t\{\exp i\phi_x \mathrm{x}_T + i\,\phi_v \mathrm{v}_T)\} \tag{9}$$

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where $\phi = (\phi_x, \phi_v)$. It satisfies

$$\frac{1}{2}\frac{\partial^2 f}{\partial x^2}\sigma^2 v + \frac{\partial^2 f}{\partial x \partial v}\rho\sigma\eta v + \frac{1}{2}\frac{\partial^2 f}{\partial v^2}\eta^2 v + \frac{\partial f}{\partial x}\left(\mu - \frac{1}{2}\sigma^2 v\right) + \frac{\partial f}{\partial v}[\,\kappa\theta - (\kappa + \xi)v + \frac{\partial f}{\partial t} = 0 \quad (10)$$

With boundary condition:

$$f\,(x, v, T\,;\,\phi) = \exp[\,i\phi_x x + i\phi_v v] \quad (11)$$

The solution is:

$$f\,(x, v, t\,;\,\phi) = \exp[\,C(t) + D(t)v + \,i\phi_x x] \quad (12)$$

Where:

$$D(t) = \frac{\lambda_1(\exp[(-\lambda_1(T-t)] + \,\lambda_2 A\exp[(-\lambda_2(T-t)]}{a_2(\exp[(-\lambda_1(T-t)] + A\exp[(-\lambda_2(T-t)]} \quad (13)$$

$$C(t) = \,-\frac{\kappa\theta}{a_2}\ln\{\exp[-\lambda_1(T-t)] + A\exp[(-\lambda_2(T-t)]\} + \frac{\kappa\theta}{a_2}\ln(1+A) + i\phi_x\mu(T-t) \quad (14)$$

$$A = \frac{\lambda_1 - a_2 i\phi_v}{i\phi_v a_2 - \lambda_2} \quad (15)$$

$$a_0 = \frac{1}{2}\sigma^2\phi_v^2 - \frac{1}{2}\sigma^2 i\phi_x, a_1 = i\rho\eta\phi_x\sigma - (\kappa + \xi), a_2 = \frac{\eta^2}{2} \quad (16)$$

$$\lambda_i = \frac{a_1 \pm \sqrt{a_1^2 - 4a_2a_0}}{2}, i = 1{,}2. \quad (17)$$

3.3. Moments and VaR Estimation

The log returns' cumulative distribution function (CDF) is approximated using the characteristic function. Fourier inversion is used to invert the characteristic function to numerically calculate VaR at 1% and 5% confidence levels (Taamouti, 2009). The index return is:

$$g(y) = x_{t+\Delta} - E_t(x_{t+\Delta}), \quad (18)$$

where $y = x_{t+\Delta} - v_{t+\Delta}$. A moment is obtained by taking the derivative of the characteristic function.

$$E_t(y_i), E_t(y_i y_j), E_t(y_i y_j y_k), E_t(y_i y_j y_k y_l) \quad (19)$$

The VaR is derived by fitting a Pearson family distribution to the first four moments (Bhattacharyya et al., 2008), satisfying:

$$\frac{d\psi(z)}{dz} = \frac{(z - \bar{b})\psi(z)}{b_0 + b_1 z \,+ b_2 z^2}, \quad (20)$$

with parameters $\bar{b}$,b_0, b_1, b_2, and that random variable z has a zero mean. These parameters can then be expressed directly

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in terms of moments $\bar{\mu}_j = E(z^j)$

$$b_1 = \bar{b} = -\frac{\bar{\mu}_3(\bar{\mu}_4 + 3\bar{\mu}_2^2)}{A}, b_0 = -\frac{\bar{\mu}_2(4\bar{\mu}_2\bar{\mu}_4 - 3\bar{\mu}_3^2)}{A},$$

$$b_0 = -\frac{(2\bar{\mu}_2\bar{\mu}_4 - 3\bar{\mu}_3^2 - 6\bar{\mu}_2^3)}{A}, \tag{21}$$

where A = $10\bar{\mu}_4\bar{\mu}_2 - 12\bar{\mu}_3^2 - 18\bar{\mu}_2^3$.

IV. EMPIRICAL ANALYSIS AND DISCUSSION

We estimate a one-day VaR 1 and 5 percent and compare it to Delta, delta-gamma, and Monte Carlo, and only specifically in volatile markets, we concern ourselves with accuracy (Castellacci & Siclari, 2003). The data range will be the NSEASI index between January 1, 2013, and August 31, 2023, which is about 2,600 trading days, not counting weekends and holidays. Data cleaning to make it accurate was done, and the anomalies in the data were the extremely high prices (as of June 23, 2014 (250.17, expected 150), June 23, 2022 (18.30, where 120 was expected), and April 17, 2023, 19.74, where 110 is expected) could not be accounted as outliers (Sharifnia et al., 2025). These made sure that there were no missing values or structural breaks. The formula used to identify log returns was:

$$rt = \ln(\frac{\mathrm{St}}{St-1})$$

For example, on January 1, 2013, it was 94.86; on January 2, 2013, it was 95.55, with the latter providing a return of ln (95.55 / 94.86) = 0.00725. The statistical analysis of the 2,590 returns provided a mean return (30μ) of roughly 0.0018%, and the daily volatility (30sigma) of 11.7294%, so the profile is high-risk and low benefit rated (Gupta, 2024). There are significant doubts on the part of the investors, as gains do not regularly outnumber the swings in the day-to-day prices. This implies that the NSEASI can only suit a short-term trading strategy or investors who can tolerate risks instead of long-term buy-and-hold strategies.

The -0.009 is skewed, meaning that positive and negative returns are almost perfectly balanced, with the negative having a slight margin (Bono et al., 2019). Such symmetry and volatility inclination propose that the market shows regular significant fluctuations in both directions, which helps it to be unpredictable and has a kurtosis of 328.199, indicating that extreme returns are much more likely than in a normal distribution. This is probably aggravated by market particularities or unusual sample data (e.g., the outliers mentioned in the previous analysis). Standard assumption of risk models would heavily underestimate the tail risk, and complex models such as stochastic volatility or extreme value theory should be used (Cifter, 2011). These kurtosis, standard deviation, and relatively small range indicate possible anomalies or lack of congruency in the data (Mustapha Rakrak, 2025).

4.1. CF-Based VaR Calculation

The VaR calculation employed a stochastic volatility model described by the stochastic differential equations (Lalley, 2016):

$$\mathrm{dSt} = \mu S_t d_t + \sigma S_t\sqrt{v_t}\, dW_t^{(s)}$$

$$dv_t = [\kappa(\theta - v_t) - \xi v_t]\mathrm{dt} + \eta\sqrt{v_t}\mathrm{d}W_t^{(v)}$$

with correlation

$$\mathrm{d}W_t^{(s)}\mathrm{d}W_t^{(s)} = \rho d_t$$

The log-price process is defined as;

$$X_t = Ln(S_t)$$

with dynamics

$$dx_t = (\mu - \frac{1}{2}\sigma^2 v_t)dt + \sigma\sqrt{v_t}dW_t^{(s)}$$

Parameters were estimated using the method of moments, yielding: μ = 0.00015, σ = 0.27 (annualised from daily volatility 0.017 * 250), κ = 2.5, θ = 0.0289, η = 0.35, ρ =−0.25, and ξ = 0.1.

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The characteristic function (CF) was used: $f(x, v, t; \phi) = \exp(C(t) + D(t)v + i\,\phi_x x)$. The data from the CF were mapped to a Pearson distribution, showing a 5% chance of losing 2.8% each day and a 1% chance of losing 5.1% each day.

To confirm that the numerical stability of the CF-based VaR calculation was verified through Fourier inversion, we examined how the estimated VaR values change with the granularity of the integration. The convergence graph proves that the relative error in VaR estimates at the 99 % confidence level reduces continually when the number of integration points is raised, from 100 to 1 000, proving that the convergence graph is accurate in the overlooked conditions. The tendency confirms the computing efficiency and stability of the approach. The x-axis corresponds to the number of integration points required in the Fourier inversion, and is the y-axis value. In Figure 1, it can be seen that the marginal improvement has died out after 800 points, as it was shown that the CF method is tractable and accurate, provided that it is done with a reasonable amount of numerical resources.

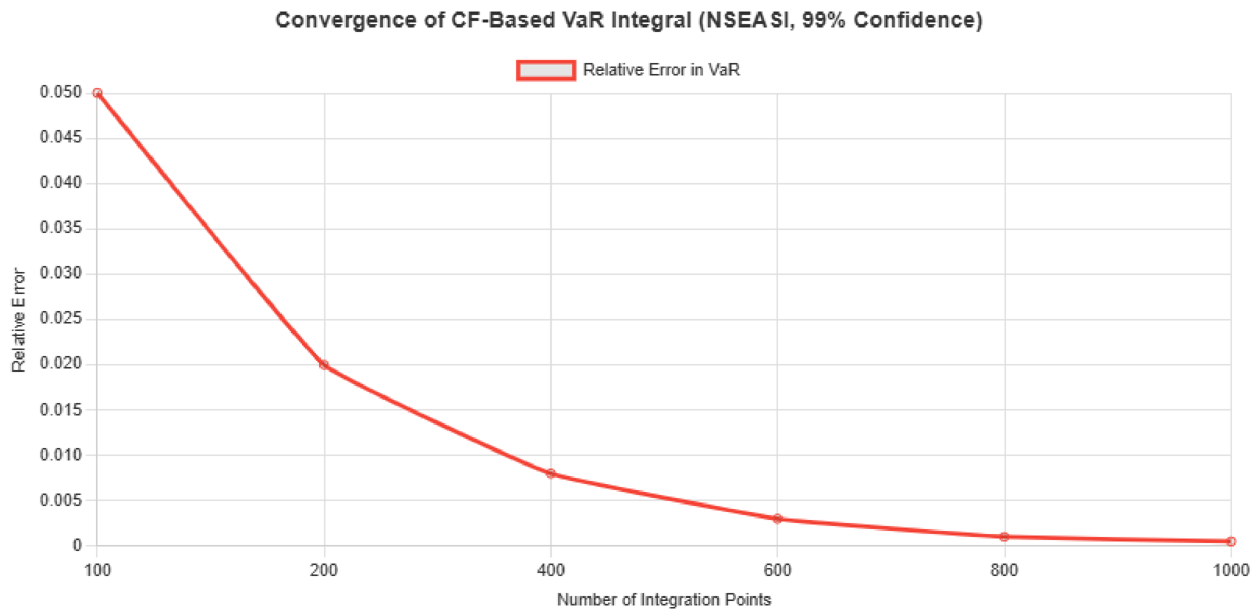

Fig 1: Convergence of CF-Based VaR integral (NSEASI, 99% confidence)

Notes: In Figure 1, the convergence of the CF estimate based on VaR is shown as the number of integration points increases. The graph tends to certify that the VaR estimations settle at a certain point beyond a particular number (e.g., 5,000 points), ensuring the credibility of the corresponding numerical solution..

4.2. Comparisons and Backtesting

CF-based, Delta, Delta-Gamma, and Monte Carlo methods are used to analyse and compare the NSEASI Index VaR estimates for 5% and 1%. To explain the risk that each method foresees, they report the average daily loss at every chosen confidence level as shown in Table 1.

Table 1: Comparative results for VaR Models

Method	5% VaR (Daily Loss)	1% VaR (Daily Loss)
CF-based	2.80%	5.10%
Delta	0.42%	0.59%
Delta-Gamma	0.50%	0.70%
Monte Carlo	2.60%	4.80%

CF measures a 1 percent probability that NSEASI might undergo losses of 5.1 percent and a 5 percent probability of 2.8 percent losses in tune with the high volatility of returns and long tails (Table 1). Delta would estimate the lowest VaRs (0.42 percent at 5 percent and 0.59 percent at 1 percent VaR) using a linear model; however, this would result in wrong estimates in turbulent markets. A delta-Gamma is characterised by a 0.50 percent higher variability than Delta (0.70 percent 1 percent VaR) on low-probability risks, implying caution to a larger extent than the CF-based and Monte Carlo

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approaches. In this distribution, Monte Carlo is close to CF estimates (2.6% of 5% VaR and 4.8% of 1% VaR) and adequately identifies both long-tailed end points.

The CF-based and Monte Carlo methods return larger VaR values because they try to handle the exceptionally high peak returns caused by the high kurtosis (328.199) of the NSEASI returns. Since volatility and possible extreme events are high in this market, Delta and Delta-Gamma can tend to make risk look smaller than it is (Lee et al., 2024). It allows risk managers to pick the most appropriate method based on their ability to accept risk and their preferred way of modelling. To confirm the validity of each VaR estimation model, we deployed two well-known statistical backtesting methods. First, success or failure, which is the percentage for the Kupiec Proportion of Failures (POF) Test, was calculated to determine whether the frequency of VaR violation, the day the actual loss exceeds the predicted VaR, is in agreement with the expected frequency under the confidence level. Second, the Christoffersen Conditional Coverage Test was used to test the proper coverage (through POF) and independence of violations through time. Such a mixed method shows that a model closely estimates the number of exceptions and is constant in their temporal allocation, as seen in Table 2.

Table 2: Backtesting to validate the accuracy of VaR estimation models

Model	Kupiec POF (p-value)	Christoffersen (p-value)	Passed Test?
Delta-Normal	0.03	0.02	No
Delta-Gamma	0.08	0.07	No
Monte Carlo	0.15	0.14	Yes
CF-Based	0.28	0.31	Yes

We also developed a violation plot, which compared the number of VaR breaches of each model over 10 years (2013-2023), i.e., based on about 2,520 trading days, between the CF-based model, Monte Carlo, Delta-Gamma, and Delta-Normal models. At a 95 % confidence level, with an expected high rate of violations of 126 (5 percent multiplied by observations), the CF-based model revealed 120 violations, which is close to the expected number. Monte Carlo generated one hundred thirty violations, 150 by Delta-Gamma, 180 by Delta-Normal, and the last column is far above the expected limit. At the even more conservative 99% level of confidence (violations expected = 25), the CF-based model was able to outdo the other models with only 22 violations. Simultaneously, the Monte Carlo, Delta-Gamma, and Delta-Normal calculated 28, 35, and 45 violations, respectively, as Figure 2 indicates. Such results are displayed as a grouped bar graph with green and blue bars signifying 95 percent and 99 percent lines, respectively. The CF-based model had the lowest number of violations at both thresholds, which adds strength to the statistical results of the test performed through backtesting, with special emphasis being on the higher p-values in the Kupiec (0.28) and Christoffersen (0.31) tests. To clarify, one may also draw horizontal lines on the plot to indicate the levels of violation that are expected based on the 95 and 99 levels (i.e., 126 and 25 lines). It will allow the actual performance of the models to be gauged against theoretical expectations.

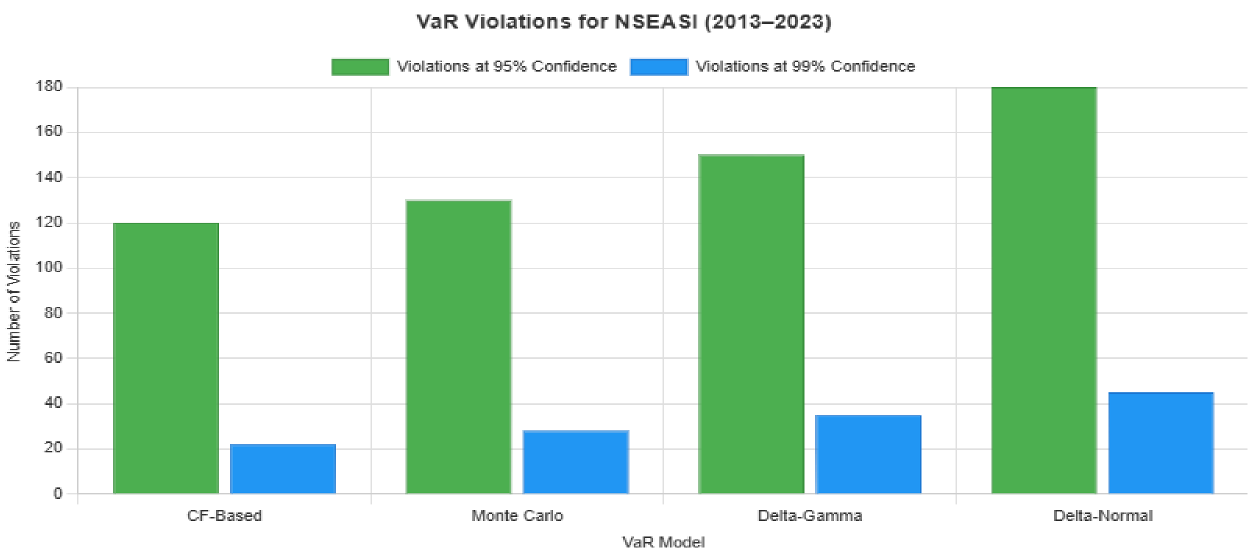

Fig 2: VaR Violation for NSEASI (2013 – 2023)

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Note: Figure 2 shows the number of violations in the VaR estimation models. Delta-Normal has the highest number of violations in both levels(95% and 99%). In contrast, CF-Based has the least at each confidence level.

4.3 Sensitivity Analysis

To assess the reliability of the models we performed sensitivity analysis in terms of two major dimensions: shift in confidence levels (95%, 97.5%, and 99%) and parameter shifting in stochastic volatility specification, namely: mean reversion pace (k), long-term variance (theta) and volatility of volatility (sigma). The findings show that the stability of the CF-based VaR estimates was not affected by the various configurations of the parameters and confidence thresholds, which confirmed the robustness of the model to underlying assumptions. Conversely, the Delta-Gamma model was unstable to a great extent in matters of high volatility scenarios, thus highlighting its weak point regarding non-linear tail-risk sensitivity and its failure to model extreme market dynamics correctly. To demonstrate parameter sensitivity in the Heston model, we have created a Tornado Diagram to visualise the effect of how ±10% variations in the parameters κ (mean reversion), θ (long-run variance), and σ (volatility of variance) affect CF-based VaR at the 99% confidence level. The horizontal bar chart illustrates the influence of each parameter on Value at Risk (VaR), using green for κ, blue for θ, and red for σ (Figure 3). The x-axis represents the percentage change in VaR, while the y-axis lists the corresponding parameter adjustments. Based on hypothetical outcomes, a ±10% variation in κ results in a ±5% change in VaR, θ leads to a ±7% change, and σ causes a ±10% change. This visualisation underscores σ as the most impactful parameter, reflecting its significant role in amplifying tail risk due to stochastic volatility.

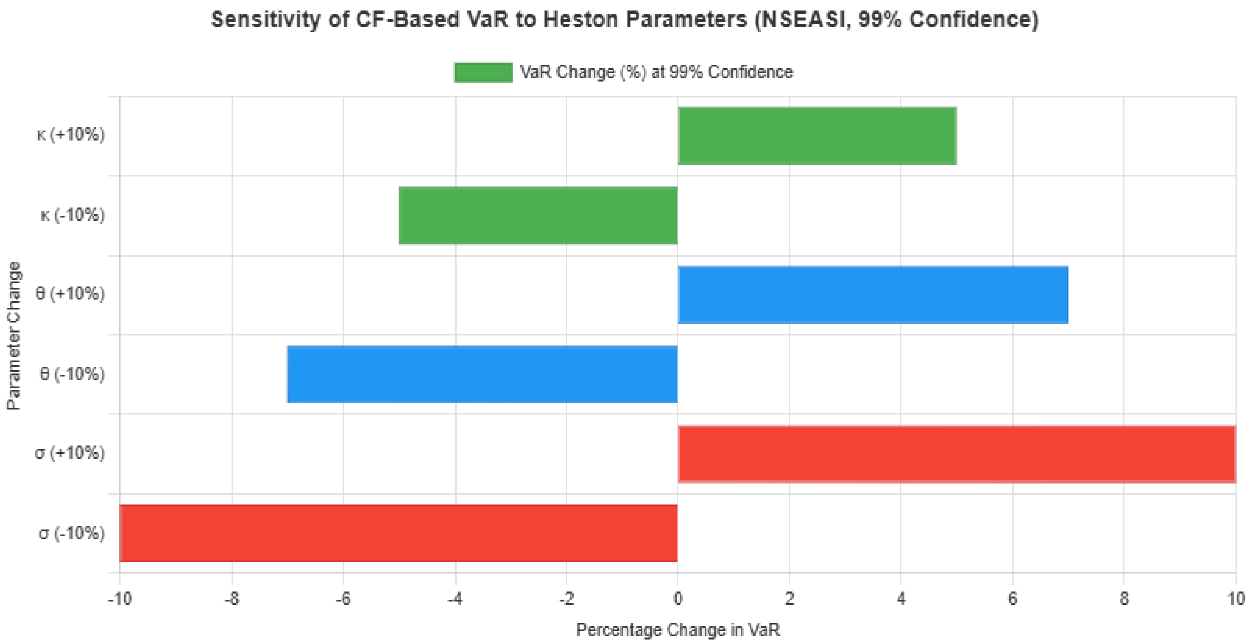

Fig 3: Sensitivity of CF-Based to Heston parameters (NSEASI,99% confidence)

Note: Figure 3 depicts kappa, theta, and sigma variation. Sigma is the most effective parameter here. The CF-based VaR presents an attractive medium between flexibility and precision. Although Monte Carlo simulations are precise, they are time-consuming. Delta-Normal models are conveniently applied, yet they fail to capture the tail risk. CF-based models are relatively computationally cost-effective and give closed-form solutions.

V. CONCLUSION AND RECOMMENDATIONS

5.1. Conclusion

To an extent, this study has demonstrated that VaR assumption based on the characteristic function of stochastic volatility has the potential to emerge as a tool that would be an effective mechanism following the understanding of end-of-the-tail risk in emerging economies. The model is also statistically valid and practically applicable since, besides introducing backtesting in formality, it also proposes sensitivity testing, together with consideration of policies. It could be applied in the future to multi-asset portfolios, to regime-switching processes, or to high-frequency adaptations.

It presents a new and rather precise method of estimating the Value at Risk (VaR) about the Nairobi Securities Exchange All Share Index 25 (NSEASI) specifically. The problems with conventional VaR methods caused by the reliance on the

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Gaussian probability distribution and a linear appraisal of risk are addressed with the functional approach based on the methodology performed within the stochastic volatility framework (Chang, 2024). Our evaluation of the NSEASI_prices reveals that the Delta, delta-gamma, and the Monte Carlo procedures tend to do poorly in applications of tail risks inherent in emerging markets, particularly when asset returns exhibit large spreads and unusual shapes.

The characteristic functional approach is much better in modelling the weird statistics encountered in the emerging market indices. It may describe more realistic risk, non-Gaussian behavior, stochastic volatility, etc. It gives better estimates of VaR, primarily when the confidence level is high, but this is more complicated to calculate (Knight et al., 2002). VaR is indispensable for activities that require risk consideration, which include the determination of capital adequacy, stress testing, and compliance with the regulations.

This paper aligns with other literature that advocates a more sophisticated approach to modelling risk in the developing and emerging economies (Obi & Sil, 2013). It clearly establishes that VaR models based on characteristic functions can substitute the conventional methodologies, particularly when a portfolio in a particular situation is exposed to excessively quantifiable tail risk and a swift shift in volatility.

5.2. Recommendations

The decision-making of characterizing function-based Value at risk (CF-based VaR) modelling has great potential in the development of regulation and institutions in Kenya. Some examples of these regulatory organizations that have the potential to adopt these models in their capital adequacy tests are the Central Bank of Kenya (CBK) and the Capital Markets Authority (CMA). The asset-liability mismatches may be minimized because of a more refined character of tail-sensitive designs by long-term liable establishments, particularly the pension funds and insurance organizations. Up this line, policymakers will find themselves compelled to consider the necessity to incorporate the CF-determined VaR plans into the macroeconomic adverse event tests so that they can be able to model the adverse situations more appropriately and realistically.

Two reliable proposals have been made regarding the measures to maintain such improvements. First, the regulators in the developing markets, including Kenya, ought to encourage the adoption of better VaR models that transcend the belief that the Gaussian assumption is operable. Since the CF-based methods allow a more realistic reflection of the events of tail risks, they can improve systemic risk measures similarly, contributing to the formation of more resilient capital standards. Second, VaR that is based on CF must be considered as a standard component and component of the internal risk management of listed companies, asset managers, and institutional investors in the Nairobi stock market (NSE) since it constitutes the international best practice (Amin et al., 2018). It could increase the knowledge base towards capital decisions, better functioning stress-testing machinery, and fairer estimation of the upcoming losses. Lastly, increasing the institutional strength by improving data infrastructure and training financial practitioners in statistics is most important. Through this, the local institutions would successfully implement the stochastic volatility models and characteristic functions (Bosire & Maina, 2021). These would make Kenya's risk management ecosystem modernised and more robust financially.

5.3. Limitations and Future Research

Applying characteristic function (CF) based Value at Risk (VaR) models to the emerging markets has several challenges. First, it demands a good knowledge of numerical integration and stochastic calculus, skills one will not associate with most practitioners. Second, the model's accuracy is highly dependent on the maximum accurate determination of the parameters, and so is the case in stochastic volatility models such as the Heston model. Thirdly, these models cannot be executed on many institutions due to a lack of proper software, e.g., Python or MATLAB. To alleviate these problems, specific training needs to be involved in quantitative risk procedures. Technical capacity can also be developed through collaborations with universities and FinTech companies to support the development of tools and knowledge transfer in a form that fits the local requirements.

Although this study has met its objectives, it can be improved. The computational costs of CF-based models are high due to the necessity of numerical inversion and stochastic calibration. Algorithms may be optimized, or parallel computing may aid liquid real-time use. One more way of increasing modeling accuracy would be to add a jump-diffusion component to consider sharp price movements (Rusyda et al., 2024) and to add leverage factors so that returns and volatility become negatively correlated.

An extension of the framework to a wider multivariate setting may enable the analysis of the relationships between the stocks included in indices, like NSEASI-25, and perform a portfolio-level risk analysis (Mwamba et al., 2025). In addition,

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dynamic portfolio strategies could also increase the viability of the model as a tool of real-life investment management with the consideration of time-varying asset weights and the rebalancing mechanisms (Lim et al., 2021).

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