IARJSET ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 8.311 ⁜ Peer-reviewed & Refereed journal ⁜ Vol. 12, Issue 7, July 2025

DOI: 10.17148/IARJSET.2025.12714

Machine Repair Systems with Heterogeneous Spares and N-Policy Constraints in Time-Shared Operations

Karunesh Kumar[1*], Manju Sharma[2]

Research Scholar, Department of Mathematics, Agra College, Agra (India), Affiliated to Dr. Bhimrao Ambedkar University Agra (U.P), India [1]

Professor, Department of Mathematics, Agra College, Agra (India), Affiliated to Dr. Bhimrao Ambedkar University Agra (U.P), India[2]

Abstract: This study represents a sophisticated framework for optimizing system performance and reliability. These systems address the challenges of maintaining operational efficiency in environments where machines are prone to failures and repair resources are limited. Heterogeneous spares, characterized by varying operational and failure rates, provide flexibility in replacing failed machines, while N-policy constraints regulate repair initiation based on the number of failed machines, ensuring efficient resource allocation. Time-shared operations further enhance repair scheduling by balancing workload distribution among repair servers. This model integrates key performance measures, such as system availability, mean failed machines, and repair server utilization, alongside cost metrics like operational, repair, and idle costs, to provide a comprehensive analysis. By leveraging these components, the system aims to minimize downtime, optimize repair efforts, and reduce overall costs, making it highly applicable in manufacturing, production, and industrial maintenance domains.

Keywords: Machine repair systems, heterogeneous spares, N-policy constraints, time-shared operations, system availability, repair server utilization, failure rates

I. INTRODUCTION

Machine repair systems play a crucial role in maintaining the efficiency and reliability of industrial and manufacturing operations, particularly in settings where equipment is prone to frequent failures. The incorporation of heterogeneous spares, characterized by varying operational and failure rates, provides a flexible and cost-effective solution for replacing failed components. Additionally, the use of N-policy constraints, which dictate that repair activities commence only when the number of failed machines reaches a predetermined threshold, ensures efficient utilization of repair resources and reduces unnecessary downtime. In time-shared operations, repair servers are dynamically allocated among failed machines, allowing for a balanced workload distribution and minimizing idle time for repair personnel. This framework integrates performance measures such as system availability, mean failed machines, and repair utilization with cost metrics like operational, repair, and idle costs. By addressing the interplay between failure, repair, and resource allocation, machine repair systems with heterogeneous spares and N-policy constraints in time-shared operations provide a robust foundation for optimizing performance and cost-effectiveness in complex industrial environments.

Jain and Preeti (2014) provided valuable insights into the integration of multiple failure types within repair models, emphasizing the need for robust systems to handle operational uncertainties effectively. **Gan et al. (2015)** emphasized balancing maintenance and inventory costs while ensuring production continuity, which set a foundation for later works focusing on integrated optimization. **Sleptchenko and van der Heijden (2016)** highlighted the benefits of integrating redundancy planning with inventory management to enhance system reliability and reduce operational costs. **Ba et al. (2016)** examined the interplay between preventive maintenance and spare parts inventory in the context of production planning. Their work extended optimization frameworks to include CO2 emissions, demonstrating the importance of incorporating environmental considerations into operational decision-making. **Olde Keizer et al. (2017)** emphasized the role of component interdependencies in optimizing maintenance schedules and inventory policies. **Wang et al. (2018)** focused on leveraging predictive models to enhance spare parts ordering decisions, contributing to the literature on proactive maintenance strategies. **Van Rooij and Scarf (2019)** provided practical insights into linking maintenance activities with production targets, ensuring an integrated approach to operational management. **Yan et al. (2020)** highlighted the challenges and trade-offs involved in optimizing multi-unit systems with varying maintenance

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efficiencies. **Ruiz et al. (2020)** underscored the importance of considering spare parts' shelf-life in inventory decisions, contributing to more sustainable inventory management practices. **Zhang et al. (2021a)** emphasized the complexity of managing spare parts with differing deterioration rates, providing actionable strategies for improving reliability. **Zhang et al. (2021b)** offered a novel perspective on leveraging system states to optimize maintenance and inventory decisions. **Zhu et al. (2022)** highlighted the unique challenges of coordinating inventory and project timelines in high-stakes operational contexts. **Abderrahmane et al. (2022)** highlighted the importance of adaptability in maintenance strategies for renewable energy systems, emphasizing the need for resilience. **Zheng et al. (2023)** provided practical solutions for enhancing system availability and reliability. **Scarf and Syntetos (2024)** synthesized advancements in the field and identified gaps for future research, making it a seminal work for practitioners and researchers.

Key contributions include the development of stochastic models that address variability in failure rates, repair times, and spare part heterogeneity, along with the implementation of N-policy constraints to balance operational efficiency and resource allocation. Despite these advancements, gaps remain in real-time decision-making, multi-objective optimization, and scalable solutions for complex systems. Additionally, the integration of environmental sustainability into these models is an emerging area of interest. Future research should focus on leveraging advanced technologies, such as machine learning and real-time analytics, to create dynamic and practical solutions that address the challenges of modern machine repair systems while ensuring environmental and operational sustainability.

II. NOTATION

$P_i(t)$: Probability of being in state iii at timett.
λ_i: Transition rate from state i to $i+1$ (failure rate).
μ_i: Transition rate from state i to $i-1$ (repair rate).
N: Threshold level (N-policy constraint).
M: Total number of machines.
S: Number of spares.
c: Number of repair servers.
t: Time variable.

III. ASSUMPTIONS

(i)The repair process follows exponential distributions.
(ii) The spares and working machines are heterogeneous, leading to variable transition rates.
(iii)Time-sharing is implemented for servers.
(iv)The system operates under the N-policy: the repair process starts only when the number of failed machines reaches N.

IV. TRANSIENT-STATE EQUATIONS

(i) State 0 (All machines are working):

$$\frac{dP_0(t)}{dt} = -M\lambda_0 P_0(t) + \mu_1 P_1(t) \tag{1}$$

(ii) State 1 (1 machine has failed, $M-1$ are operational):

$$\frac{dP_1(t)}{dt} = M\lambda_0 P_0(t) - (\lambda_1 + \mu_1)P_1(t) + 2\mu_2 P_2(t) \tag{2}$$

(iii) State 2 (2 machines have failed, $M-2$ are operational):

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2 + \mu_2)P_2(t) + 3\mu_3 P_3(t) \tag{3}$$

(iv) State i (i machines have failed, $M-i$ are operational):

$$\frac{dP_i(t)}{dt} = (M-i+1)\lambda_{i-1}P_{i-1}(t) - [(M-i)\lambda_i + i\mu_i]P_i(t) + (i+1)\mu_{i+1}P_{i+1}(t) \tag{4}$$

(v) State $N-1$ (1 machine short of triggering N-policy):

$$\frac{dP_{N-1}(t)}{dt} = (M-N+2)\lambda_{N-2}P_{N-2}(t) - [(M-N+1)\lambda_{N-1} + (N-1)\mu_{N-1}]P_{N-1}(t) + N\mu_N P_N(t) \tag{5}$$

(vi) State N (Repair process starts):

$$\frac{dP_N(t)}{dt} = (M-N+1)\lambda_{N-1}P_{N-1}(t) - (M-N)\lambda_N P_N(t) \tag{6}$$

(vii) State $N+1$ (Repair ongoing with $N+1$ failed machines):

DOI: 10.17148/IARJSET.2025.12714

$$\frac{dP_{N+1}(t)}{dt} = (M-N)\lambda_N P_N(t) - (M-N-1)\lambda_{N+1}P_{N+1}(t) \quad (7)$$

(viii)State $M-1$ (All but one machine have failed):

$$\frac{dP_{M-1}(t)}{dt} = \lambda_{M-2}P_{M-2}(t) - \mu_{M-1}P_{M-1}(t) \quad (8)$$

(ix) State M (All machines have failed): (9)

$$\frac{dP_M(t)}{dt} = \lambda_{M-1}P_{M-1}(t)$$

(x) State with Spare Allocation: For $i < S$

$$\frac{dP_i(t)}{dt} = \lambda_i P_i(t) + \mu_i P_{i+1}(t) \quad (10)$$

V. MATRIX GEOMETRIC METHOD (MGM) FOR THE SOLUTION OF PROPOSED MODEL

The Matrix Geometric Method is a powerful approach to analyze systems with repetitive or block-structured Markov processes, particularly in quasi-birth-death (QBD) processes. For the machine repair system with N-policy constraints, the states exhibit a block structure because the transitions depend on the number of failed machines, making the system amenable to this method.

The state space can be grouped into levels based on the number of failed machines. Each level i represents a set of states where iii machines have failed, and the repair or failure rates depend on i.

The transition rate matrix Q is expressed in a block form:

$$Q = \begin{bmatrix} B_0 & A_1 & 0 & \dots & 0 \\ C_1 & B_1 & A_2 & \dots & 0 \\ 0 & C_2 & B_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & B_M \end{bmatrix} \quad (11)$$

A_i: Block representing transitions from level i to level $i+1$ (failures).
B_i: Block representing transitions within level i (repair or failure).
C_i: Block representing transitions from level i to level $i-1$ (repairs).
The steady-state probability vector Π is partitioned into level probabilities:

$$\Pi = [\Pi_0, \Pi_1, \dots \Pi_M] \quad (12)$$

where Π_i represents the probability distribution within level i.

The balance equations for each level are:

$$\text{Level 0: } \Pi_0 B_0 + \Pi_1 C_1 = 0 \quad (13)$$

$$\text{Level i } (1 \le i \le M-1): \Pi_i B_i + \Pi_{i-1}A_i + \Pi_{i+1}C_{i+1} = 0 \quad (14)$$

$$\text{Level M: } \Pi_M B_M + \Pi_{M-1}A_M = 0 \quad (15)$$

Assume a geometric structure for Π_i

$$\Pi_i = \Pi_0 R^i, \text{for } i \ge 1 \quad (16)$$

where R is the rate matrix, determined by solving the matrix quadratic equation:

$$C_i + RB_i + R^2 A_i = 0 \quad (17)$$

Once R is computed, the probabilities for each level are recursively obtained: $\Pi_i = \Pi_0 R^i$

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The steady-state probabilities must satisfy: $\sum_{i=0}^{M} \Pi_i = 1$ (18)

VI. IMPLEMENTATION OF MGM

We have $B_i = -(\lambda_i + \mu_i),\ A_i = \lambda_i, C_i = \mu_i$ (19)

The rate matrix R satisfies the equation (17):

$\mu_i - R(\lambda_i + \mu_i) + R^2\lambda_i = 0 \Rightarrow R^2\lambda_i - R(\lambda_i + \mu_i) + \mu_i = 0$ (20)

Solve for R using the quadratic formula:

$R = \frac{(\lambda_i+\mu_i)\pm\sqrt{(\lambda_i+\mu_i)^2-4\lambda_i\mu_i}}{2\lambda_I}$ (21)

Choose the root where $|R| < 1$, as probabilities must be bounded.

Using equation (16), we get

$\Pi_1 = \Pi_0 R$

$\Pi_2 = \Pi_0 R^2$

$\Pi_3 = \Pi_0 R^3$

$\dots \quad \dots \dots$

$\Pi_M = \Pi_0 R^M$

Since we have $\sum_{i=0}^{M} \Pi_i = 1$

$\Rightarrow \Pi_0(R + R^2 + R^3 + \cdots R^M) = 1 \Rightarrow \Pi_0 \frac{1-R^{M+1}}{1-R} = 1 \Rightarrow \Pi_0 = \frac{1-R}{1-R^{M+1}}, |R| < 1$ (22)

Table 1: State Probabilities for the proposed Machine system

State (i)	Failure Rate (λ_i)	Repair Rate (μ_i)	State Probability (Π_i)
0	0.5	0.3	0.502495887
1	0.375	0.4	0.259217952
2	0.25	0.5	0.133720391
3	0.125	0.6	0.068981114
4	0	0.7	0.035584656

VII. PERFORMANCE MEASURES

For a machine repair system, the following performance measures provide insights into the system's effectiveness and efficiency:

(i) System Availability (A)**:** The probability that a machine is operational:

$A = \sum_{i=0}^{M}(M - i)\frac{\Pi_i}{M}$ (23)

where $(M - i)$ represents the number of operational machines in state i, and Π_i is the probability of being in state i.

(ii) System Unavailability (U): The complement of system availability, representing the probability that a machine is not operational: $U = 1 - A$

(iii) Mean Number of Failed Machines (F): The expected number of failed machines:

$\mathbf{F = \sum_{i=0}^{M} i}\ \Pi_i$ (24)

(iv) Mean Number of Machines Under Repair $(\mathbf{R_{Mean}})$**:** The average number of machines being repaired:

$R_{mean} = \sum_{i=1}^{M} \min(i, c). \Pi_i$ (25)

where $\min(i, c)$ ensures that the number of repairs does not exceed the number of repair servers (R).

(v) Idle Probability of Repair Servers (P_{ideal})**:** The probability that a repair server is idle:

$P_{idle} = \sum_{i=0}^{M}[R - \min(i, c)]\frac{\Pi_i}{c}$ (26)

(vi) Repair Utilization (U_r)**:** The fraction of time the repair servers are busy: $U_r = 1 - \mathbf{P_{ideal}}$

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Table 2: Performance Measures of the proposed Machine Repair System

Measure	Value
System Availability (A)	0.781014825
System Unavailability (U)	0.218985175
Mean Number of Failed Machines (F)	0.875940699
Mean Number of Machines Under Repair (R_{mean})	0.840356043
Idle Probability of Repair Servers (P_{ideal})	0.719881319
Repair Utilization (U_r)	0.280118681

VIII. RELIABILITY ANALYSIS

Reliability $\mathbb{R}(t)$ of the machine repair system is the probability that a machine functions without failure for a specified time ttt. Reliability is closely tied to the failure rate (λ_i) and repair dynamics of the system.

The system reliability is the weighted average of the reliability across all states:

$$\mathbb{R}_{system}(t) = \sum_{i=0}^{M} \Pi_i \mathbb{R}_i(t) \tag{27}$$

where: $\mathbb{R}_i(t) = e^{-\lambda_i t}$

and Π_i is the steady-state probability of being in state i.
We compute the system reliability $\mathbb{R}_{system}(t)$ at specific time points = [0.5,1,1.5,2] , using the failure rates (λ_i) and probabilities (Π_i).

Table 3: System Reliability

Time (t)	System Reliability $R_{system}(t)$
0.5	0.824637667
1	0.683538702
1.5	0.569736786
2	0.477716515

IX. COST ANALYSIS

To perform a cost analysis of the machine repair system, we need to consider different cost components that are typically involved:

(i) Operational Cost: Cost incurred when machines are working.
(ii) Failure Cost: Cost incurred due to machine failures.
(iii)Repair Cost: Cost associated with repairing failed machines.
(iv) Idle Cost: Cost due to idle repair servers when there are no failed machines to repair.
C_O: Cost per operational machine per unit time.
C_f: Cost per failed machine per unit time.
C_r: Repair cost per unit time per repair server.
C_i: Idle cost per unit time per idle repair server.

The total cost (C_{Total}) is the sum of all the components:

$$C_{Total} = C_O.\text{Operational Cost} + C_f.\text{Failure Cost} + C_r.\text{Repair cost} + C_i.\text{Idle Cost}$$
$$\text{Operational Cost} = \sum_{i=0}^{M}(M-i)\Pi_i \tag{28}$$

where $M - i$ is the number of operational machines in state i, and Π_i is the probability of being in state i.
$$\text{Failure Cost} = \sum_{i=0}^{M} i.\Pi_i \tag{29}$$

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Repair Cost $= \sum_{i=1}^{M} \min(i, c) \,.\, C_r . \Pi_i$
where c is the number of repair servers, and $\min(i.c)$ accounts for cases where fewer than c machines are under repair.
Idle Cost $= \sum_{i=0}^{M} \min(i.c)\, [c - \min(i.c)]. C_r. \Pi_i$ (30)

We minimize the total cost with respect to the following:
μ_i: Increasing repair rates reduces failure costs but increases repair costs.
c: Adding more repair servers decreases idle cost and failure cost but increases repair costs.
λ_i: Failure rates depend on system design and maintenance practices.

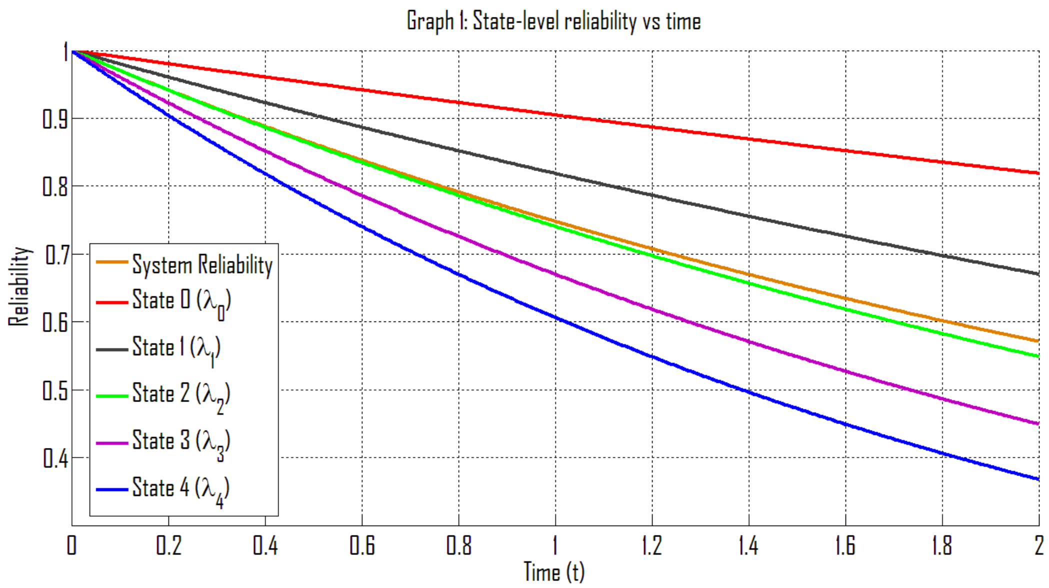
Include realistic constraints, such as:
$\mu_i > 0$: Repair rates must be positive.
$\lambda_i > 0$: Failure rates must be positive.
$c \geq 1$: At least one repair server must be present.

Let's assign numerical values to the cost parameters:
$C_O = 10$: Cost per operational machine per unit time.
$C_f = 15$: Cost per failed machine per unit time.
$C_r = 20$: Repair cost per repair server per unit time.
$C_i = 5$: Idle cost per idle repair server per unit time.
$c = 1$: Single repair server.

Table 4: Cost Analysis

Cost Component	Value
Operational Cost	31.24059301
Failure Cost	13.13911049
Repair Cost	9.95008226
Idle Cost	2.512479435
Total Cost	56.84226519

X. RESULTS AND DISCUSSION



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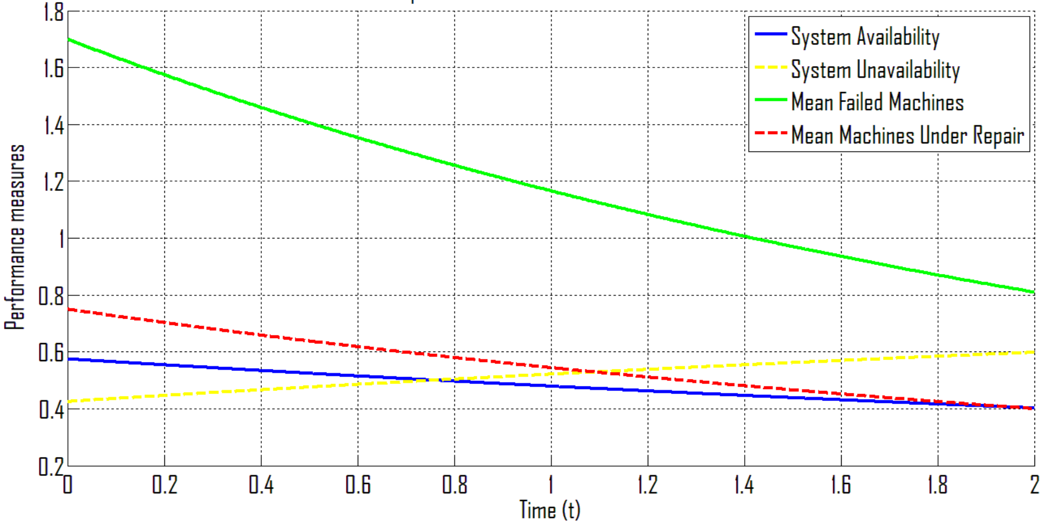
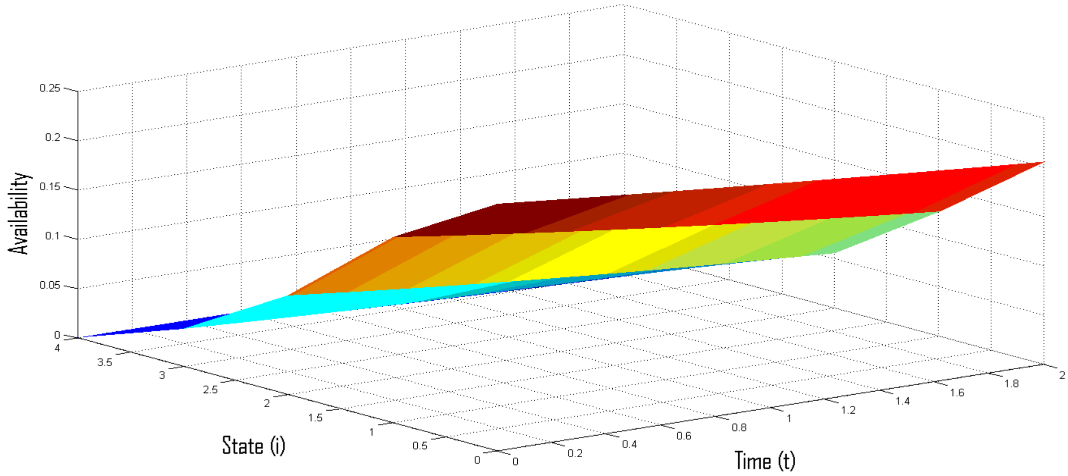
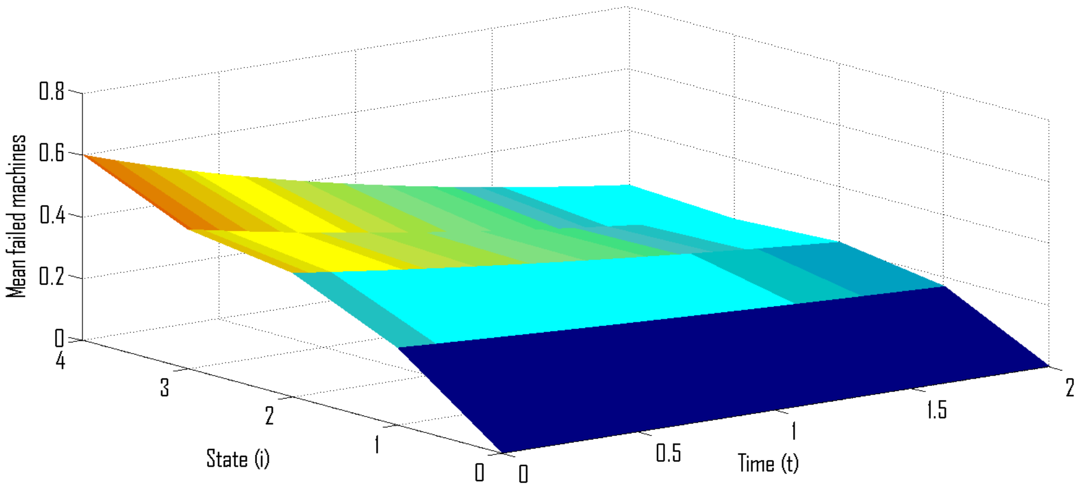
Graph 2: Performance measures vs time



Graph 3: System availability surface plot



Graph 4: Mean failed machines surface plot



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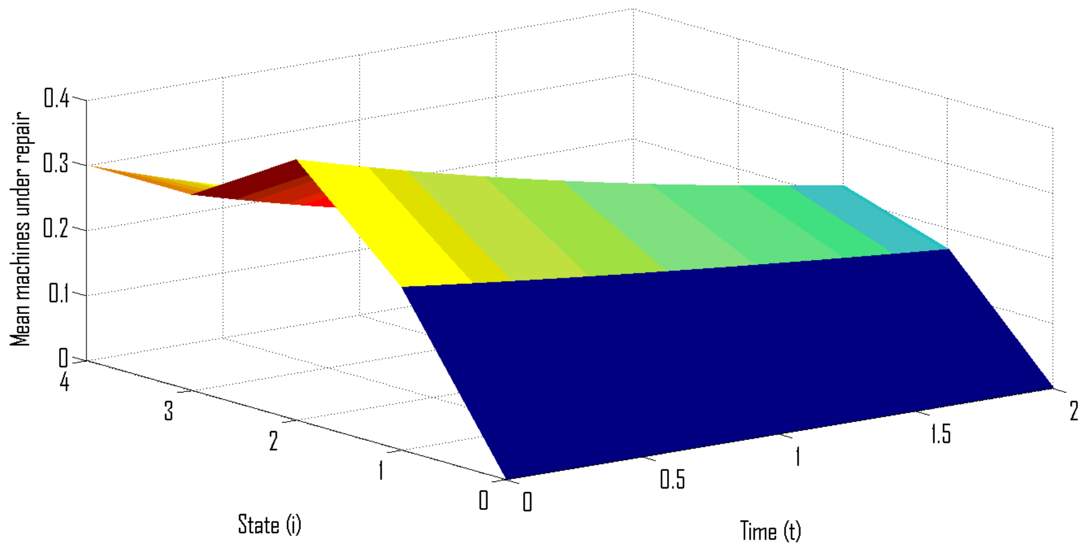
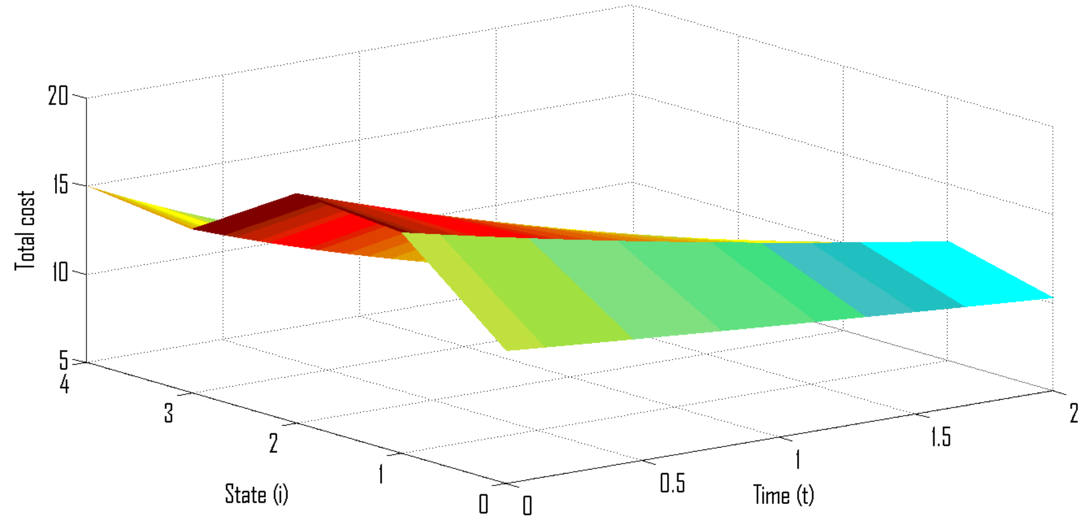
Graph 5: Mean machines under repair surface plot



Graph 6: Total cost surface plot



The graph (1) illustrates the state-level reliability and the overall system reliability as functions of time. The reliability curves for each state $(\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ represent the probability that components in each state operate without failure over time. The states with lower failure rates (λ) demonstrate slower decay in reliability, as seen in the curves for states like λ_0 and λ_1 , which remain closer to 1 for longer durations. Conversely, states with higher failure rates $(\lambda_3$ and $\lambda_4)$ exhibit faster reliability degradation. The system reliability curve combines these individual reliabilities into a weighted average, showing the overall probability of system operation. This composite curve decays less steeply than higher λ states due to contributions from the more reliable states. The graph effectively visualizes how individual component reliabilities influence the overall system reliability over time.

The graph (2) presents four key performance measures system availability, system unavailability, mean failed machines and mean machines under repair as functions of time. system availability (blue curve) decreases steadily over time as components fail, reflecting the diminishing operational capacity of the system. Conversely, System Unavailability (yellow dashed curve) increases over time, highlighting the growing probability of system downtime due to failures. The Mean Failed Machines (green curve) starts high and declines over time, representing the combined effect of failure and repair rates on the number of machines out of service. Lastly, the mean machines under repair (red dashed curve) also decreases gradually, indicating the system's declining ability to utilize repair resources as fewer failures occur over time. This graph provides a comprehensive view of how these performance measures evolve, revealing the interplay between failure, repair, and system efficiency.

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The 3D surface plot in graph (3) illustrates the system availability as a function of both time (t) and state (i). The horizontal axes represent the states (i, corresponding to the number of failed machines) and time (t), while the vertical axis displays the availability values. The plot shows how availability changes across different states and over time. Initially, for lower time values, availability is higher for states with fewer failures, as expected. As time a progress, availability diminishes across all states, with a more significant decline for states representing higher numbers of failed machines. This visualization highlights the dynamic relationship between time, system state, and availability, emphasizing how system performance degrades due to the combined effects of failure and time-dependent operational factors.

The 3D surface plot in graph (4) depicts the mean number of failed machines as a function of time (t) and state (i). The horizontal axes represent the states (iii, corresponding to the number of failed machines) and time (t), while the vertical axis shows the mean number of failed machines. At initial time values, the mean number of failed machines is higher for states with more failures. Over time, the mean number of failed machines decreases for all states, reflecting the repair processes and the system's ability to recover. States with fewer initial failures exhibit a more significant reduction in failed machines, as seen in the sharper decline near lower states. This visualization highlights the temporal evolution of machine failures and their dependence on system state and time. It effectively conveys the interplay between system degradation and recovery dynamics.

The 3D surface plot in graph (5) represents the mean number of machines under repair as a function **of** time (t) and state (i)**.** The horizontal axes indicate the number of failed machines (i) and time (t), while the vertical axis shows the average number of machines undergoing repair. Initially, for states with a larger number of failed machines $(\mathrm{i} > 2)$, the mean number of machines under repair is higher. However, as time progresses, the repair process reduces the average number of machines being repaired across all states. The decline is more prominent in states with fewer failed machines, as the system effectively addresses smaller repair loads. The plot highlights the relationship between repair activity, failure states, and time, showing how the repair process stabilizes the system by gradually reducing the workload over time.

The 3D surface plot in graph (6) illustrates the total cost as a function of time (t) and state (i). The horizontal axes represent the number of failed machines (i) and time (t), while the vertical axis displays the total cost, which includes operational, failure, repair, and idle costs. At the initial stages, the total cost is higher for states with a greater number of failed machines $(\mathrm{i} > 2)$ due to increased failure and repair activities. As time progresses, the total cost decreases across all states, reflecting the system's stabilization and reduction in failures over time. The decline in cost is more pronounced in states with fewer failures, as these states recover more quickly. This visualization effectively demonstrates the temporal dynamics of system costs and highlights the significant impact of failure and repair processes on overall expenditures.

XI. CONCLUDING REMARKS

In conclusion, this research provides a robust and efficient framework for managing machine failures in complex industrial settings. By integrating heterogeneous spares, these systems enhance operational flexibility and ensure quicker recovery from failures, while the N-policy constraints optimize repair initiation, reducing unnecessary resource utilization and downtime. The inclusion of time-shared operations further enhances the system's efficiency by balancing repair workloads and minimizing idle time for repair servers. Performance measures such as system availability, mean failed machines, and repair server utilization, coupled with cost metrics like operational, repair, and idle costs, enable a comprehensive analysis and informed decision-making. These systems demonstrate significant potential for minimizing downtime, reducing costs, and ensuring sustainable performance in industrial and manufacturing operations, making them an indispensable tool for modern reliability and maintenance engineering.

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DOI: 10.17148/IARJSET.2025.12714

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