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Some Results on a New Subclass of p-Valent Functions

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Abstract: The theory of p-valent functions is an important subject in the geometric function theory. Recently, many researchers have shown great interests in the study of p-valent functions. The aim of this paper is to investigate several results concerning the subordination of multivalent functions in the open unit disc $\mathbb{U}$; which are associated with derivative operator $\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)$.

Keywords: analytic functions, multivalent functions, differential operator, subordination.

I. INTRODUCTION

Let $\mathcal{H}(\mathbb{U})$ denote the class of analytic functions in the open unit disc $\mathbb{U} = \{z: z \in \mathbb{C}, |z| < 1\}$, and let $\mathcal{H}[a,p]$ be the subclass of $\mathcal{H}(\mathbb{U})$ of the form

$$f(z) = a + a_p z^n + a_{p+1} z^{p+1} + \cdots, \ (z \in \mathbb{U}, p \in \mathbb{N}).$$

Let $\mathcal{A}_p$ be the subclass of $\mathcal{H}(\mathbb{U})$ of the form

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n, \qquad (z \in \mathbb{U}, p \in \mathbb{N}). \tag{1.1}$$

For $f(z)$ and $g(z)$ are analytic in $\mathbb{U}$, we say that f is subordinate to g if there exists an analytic function ω in $\mathbb{U}$, with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z)), z \in \mathbb{U}$. We denote this subordination by $f(z) \prec g(z)$. If $g(z)$ is univalent in $\mathbb{U}$, then the subordination is equivalent to $f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$.

Definition 1.1. *For a function $f \in \mathcal{A}_p$ given by (1.1), we define the derivative operator by $\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)$*

$$\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z) = z^p + \sum_{n=p+1}^{\infty} \left(\frac{n}{p}\right)^{\alpha} [\beta(n-p)(\lambda-\delta)+p]^k a_n z^n, \ (z \in \mathbb{U}), \tag{1.2}$$

where $\delta \geq 0, \ \beta > 0, \ \lambda > 0, \ \delta \neq \lambda, k, \alpha \in \mathbb{N}_0 = \{0,1,2,\ldots\}$ and $p \in \mathbb{N}$.

Remark 1.1. *It should be remarked that the differential operator $\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)$ is a generalization of many operators considered earlier. Let us see some of the examples:*
For $\beta = \lambda = p = 1$ and $\alpha = \delta = 0$, we get the operator introduced by Sălăgean [5].
For $\lambda = p = 1$ and $\alpha = \delta = 0$, we get the generalized Sălăgean derivative operator introduced by Al-Oboudi [4].
For $p = 1$ and $\alpha = 0$, we obtain the operator introduced by Darus and Ibrahim [6].

The following Lemmas will be required in our investigation:

Lemma 1.1. *(see [11]) Let $q(z)$ be convex univalent in the unit disc U and let $\psi \in \mathbb{C}$ and $\gamma \in \mathbb{C} - \{0\}$ with*

$$\Re\{1 + \frac{zq''(z)}{q'(z)} + \frac{\psi}{\gamma}\} > 0.$$

If $p(z)$ is analytic in U and $\psi p(z) + \gamma z p'(z) \prec \psi q(z) + \gamma z q'(z)$, then $p(z) \prec q(z), \ (z \in U)$ and q is the best dominant.

Lemma 1.2. *(see [8]) Let $q(z)$ be univalent in the unit disc U and θ and ϕ be analytic in a domain D containing $q(U)$ with $\phi(w) \neq 0$ when $w \in q(U)$.*
Set

$$Q(z) := zq'(z)\phi(q(z)), \ and \quad h(z) := \theta(q(z)) + Q(z).$$

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Suppose that $Q(z)$ is starlike univalent in U, and $\Re\{\frac{zh\prime(z)}{Q(z)}\} > 0$ for $z \in U$. If the subordination $\theta(p(z)) + zp'(z)\phi(p(z)) \prec \theta(q(z)) + zq'(z)\phi(q(z))$ holds then $p(z) \prec q(z)$, $(z \in U)$ and $q(z)$ is the best dominant.

Lemma 1.3. *(see [9]) Let $q(z)$ be convex univalent in the unit disc U and $\gamma \in \mathbb{C}$. Further, assume that $\Re\{\gamma\} > 0$. If $p(z) \in \mathcal{H}[q(0),1] \cap Q$, with $p(z) + \gamma zp'(z)$ is univalent in U, then $q(z) + \gamma zq'(z) \prec p(z) + \gamma zp'(z)$ implies $q(z) \prec p(z)$ and $q(z)$ is the best subordinant.*

Lemma 1.4. *(see [10]) Let $q(z)$ be convex univalent in the unit disc U and ϑ and φ be analytic in a domain D containing $q(U)$. Suppose that $zq'(z)\varphi(q(z))$ is starlike univalent in U, and $\Re\{\frac{\vartheta\prime(q(z))}{\varphi(q(z))}\} > 0$ for $z \in U$. If $p(z) \in \mathcal{H}[q(0),1] \cap Q$, with $p(U) \subseteq D$ and $\vartheta(p(z)) + zp'(z)\varphi(z)$ is univalent in U and $\vartheta(q(z)) + zq'(z)\varphi(q(z)) \prec \vartheta(p(z)) + zp'(z)\varphi(p(z))$ then $q(z) \prec p(z), (z \in U)$ and $q(z)$ is the best subordinant.*

II. MAIN RESULTS

We study the subordination for functions containing derivative operator, and followed by some sandwich results.

Theorem 2.1. *Let $f, g \in \mathcal{A}_p$, $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$ be a convex univalent in the unit disc U and $\Omega, \gamma > 0$, such that $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$ be analytic in U satisfies*

$$\Re\{1 + \frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g\prime\prime(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g\prime(z)} + (\Omega - 1)\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g\prime(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} + \frac{1}{\gamma}\} > 0,$$

$$\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z) \neq 0, \mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z) \neq 0, \ z \in U.$$

If $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \in \mathcal{A}_p$ and the subordination

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1 + \Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f\prime(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}] \prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}[1 + \Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g\prime(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}],$$

holds then

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$$

and $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$ is the best dominant.

Proof. Our aim is to apply Lemma 1.1. Setting

$$p(z) := (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \ \ and \ \ q(z) := (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}.$$

It suffices to prove

$$\Re\{1 + \frac{zq\prime\prime(z)}{q\prime(z)} + \frac{1}{\gamma}\} > 0, \quad \gamma \neq 0.$$

By the assumptions of the theorem, and using the following

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))' = \mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z), \ (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))' = \mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z),$$

and

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z))' = \mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z).$$

Then $\quad \Re\left\{1 + \frac{zq''(z)}{q'(z)} + \frac{1}{\gamma}\right\}$

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$$= \Re\{1 + \frac{z\Omega(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z))^{\Omega}[\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)} - (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)})^2 + \Omega(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)})^2]}{\Omega(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z))^{\Omega}(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)})} + \frac{1}{\gamma}\}$$

$$= \Re\{1 + \frac{\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)} - z(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)})^2 + \Omega z(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)})^2}{(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)})} + \frac{1}{\gamma}\}$$

$$= \Re\{1 + \frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)} + (\Omega - 1)\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)} + \frac{1}{\gamma}\}$$
$$> 0$$

Now we show that

$$p(z) + \gamma z p'(z) \prec q(z) + \gamma z q'(z),$$

where $\Re\{\overline{\gamma}\} > 0$ and $\psi = 1$. By using the assumption of the theorem we have

$$\begin{aligned} p(z) + \gamma z p'(z) &= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega} + \gamma z[(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega}]' \\ &= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega} + \gamma z[\Omega\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}(f(z))^{\Omega-1}\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f'(z)] \\ &= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega} + \gamma\Omega[(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega}\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z)}] \\ &= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega}[1 + \Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z)}] \\ &\prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z))^{\Omega}[1 + \Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)}] \\ &= q(z) + \gamma z q'(z). \end{aligned}$$

Thus in view of Lemma 1.1, $p(z) \prec q(z)$ and q is the best dominant.

Taking $q(z) = \frac{1+Az}{1+Bz}$ in Theorem 2.1, we have the following corollary.

Corollary 2.1. *Let $q(z)$ be a convex univalent in the unit disc U and $1 \le B < A \le 1, \Omega, \gamma > 0$, such that $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z))^{\Omega}$ be analytic in U satisfies*

$$\Re\{1 + \frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)} + (\Omega - 1)\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z)} + \frac{1}{\gamma}\} > 0,$$

$$\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g(z) \neq 0, \mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}g'(z) \neq 0, \ z \in U.$$

If $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega} \in \mathcal{A}_p$ and the subordination

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega}[1 + \Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z)}] \prec \frac{1+Az}{1+Bz} + \gamma z\frac{(A-B)}{(1+Bz)^2},$$

holds then

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\ell,p}f(z))^{\Omega} \prec \frac{1+Az}{1+Bz}$$

and $\frac{1+Az}{1+Bz}$ is the best dominant.

Further taking $A = 1, B = -1$ in Corollary 2.1, we state an interesting result in the following corollary.

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Corollary 2.2. *Let $q(z)$ be a convex univalent in the unit disc U and $\Omega,\gamma > 0$, such that $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$ be analytic in U satisfies*

$$\Re\{1 + \frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)} + (\Omega - 1)\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} + \frac{1}{\gamma}\} > 0,$$

$$\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z) \neq 0, \mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z) \neq 0,\ \ z \in U.$$

If $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \in \mathcal{A}_p$ and the subordination

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1 + \Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}] \prec \frac{1+z}{1-z} + \frac{2\gamma z}{(1-z)^2},$$

holds then

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \prec \frac{1+z}{1-z}$$

and $\frac{1+z}{1-z}$ is the best dominant.

Theorem 2.2. *Let $f,g \in \mathcal{A}_p$ and $z[(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}]'$ be starlike univalent function in U. Assume that*

$$\rho(z) = \frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - p,\ \ (z \in U),$$

such that

$$\Re\{\frac{(\rho(z)+p)[\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)} - (\rho(z)+p)] + p}{\rho(z)} + \mu\rho(z) + 2\} > 0,\ \ (z \in U).$$

If $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu} \in \mathcal{A}_p$ and the subordination

$$(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z})^{\mu}\{1 + \mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)} - p)\} \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}\{1 + \mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - p)\}$$

holds then

$$(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu} \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu},\ \ \mu \geq 1,$$

and $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}$ is the best dominant.

Proof. Our aim is to apply Lemma 1.2. Setting

$$p(z) := (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}\ \ and\ \ q(z) := (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}.$$

Then we obtain

$$\begin{aligned} q'(z) &= \mu(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu-1}[\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{z^p} - \frac{p\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^{p+1}}] \\ &= \mu(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}[\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - \frac{p}{z}] \\ &= \mu q(z)[\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - \frac{p}{z}] \end{aligned}$$

and

$$\begin{aligned} q''(z) &= \mu\{q(z)(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z) - (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z))^2}{(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^2} + \frac{p}{z^2}) \\ &+\mu q'(z)(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - \frac{p}{z})\} \\ &= \mu\{q(z)[\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)})^2 + \frac{p}{z^2}] + (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)} - \frac{p}{z})q'(z)\}. \end{aligned}$$

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By letting

$$\theta(\omega):=\omega \quad and \quad \phi(\omega):=1,$$

it is clear that $\theta(z),\phi(z)$ are analytic in $\mathbb{C}$. Also, we consider

$$Q(z):=zq'(z)\phi(z)=zq'(z),$$

$$h(z):=\theta(q(z))+Q(z)=q(z)+zq'(z)$$

implies

$$h'(z)=2q'(z)+zq''(z).$$

By the assumptions of the theorem, we find that $Q(z)$ is starlike univalent in U and that

$$\Re\{\frac{zh'(z)}{Q(z)}\}=\Re\{2+\frac{zq''(z)}{q'(z)}\}$$

$$=\Re\{\frac{\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-z(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)})^2+\frac{p}{z}+\mu z(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-\frac{p}{z})^2}{\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-\frac{p}{z}}+2\}$$

$$=\Re\{\frac{\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}[\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}-\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}]+\frac{p}{z}}{\frac{1}{z}(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-p)}+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-p)+2\}$$

$$=\Re\{\frac{\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}[\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}-\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}]+p}{(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-p)}+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-p)+2\}$$

$$=\Re\{\frac{(\rho(z)+p)[\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}-(\rho(z)+p)]+p}{\rho(z)}+\mu\rho(z)+2\}$$

$$>0.$$

Now we proceed to prove

$$p(z)+zp'(z)\prec q(z)+zq'(z).$$

A computation shows that

$$p(z)+zp'(z)=(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^\mu+z[(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^\mu]'$$

$$=(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^\mu\{1+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}-p)\}$$

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$$\prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z})^{\mu}\{1+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-p)\}$$

$$= q(z) + zq'(z)$$

Thus in view of Lemma 1.2, $p(z) \prec q(z)$ and q is the best dominant.

Theorem 2.3. *Let $f, g \in \mathcal{A}_p$, $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$ be convex univalent in U and $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \in \mathcal{H}[0,1] \cap Q$. Assume that $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1+\gamma\Omega\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}]$ is univalent in U where $\Omega, \gamma \in \mathbb{C}, \Re\{\gamma\} > 0$. If $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \in \mathcal{A}_p$ and the subordination*

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}] \prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}],$$

holds then

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega} \prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}$$

and $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}$ is the best subordinant.

Proof. Our aim is to apply Lemma 1.3. Assuming that

$$p(z) := (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega} \;\; and \;\; q(z) := (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}.$$

$$\begin{aligned}
q(z)+\gamma zq'(z) &= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega} + \gamma z[(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}]' \\
&= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega} + \gamma z[\Omega(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega-1}.(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z))] \\
&= (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}] \\
&\prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}] \\
&= p(z)+\gamma zp'(z).
\end{aligned}$$

Hence in view of Lemma 1.3, $q(z) \prec p(z)$ and $q(z)$ is the best subordinant.

Theorem 2.4. *Let $f, g \in \mathcal{A}_p$ and $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}$ be convex univalent in U. Let the following assumptions satisfy:*

(i) $z[(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}]'$ *is starlike univalent function in U,*

(ii) $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}\left\{1+\mu\left(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}-p\right)\right\}$ *is univalent in U,*

(iii) $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu} \in \mathcal{H}[0,1] \cap Q$.

If $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu} \in \mathcal{A}_p$ and the subordination

$$(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}\{1+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}-p)\} \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}\{1+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}-p)\}$$

holds then

$$(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu} \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}, \;\; \mu > 1,$$

and $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}$ is the best subordinant.

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Proof. Our aim is to apply Lemma 1.4. Letting

$$p(z):=(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu} \quad and \quad q(z):=(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}.$$

By taking

$$\vartheta(\omega):=\omega \quad and \quad \varphi(\omega):=1,$$

it can easily observed that $\vartheta(z),\varphi(z)$ are analytic in $\mathbb{C}$. Thus

$$\Re\{\frac{\vartheta'(q(z))}{\varphi(q(z))}\}=1>0.$$

Now we must show that

$$q(z)+zq'(z)\prec p(z)+zp'(z).$$

A computation shows that

$$\begin{aligned} q(z)+zq'(z) &= (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}+z[(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}]' \\ &= (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g(z)}{z^p})^{\mu}\{1+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g'(z)}{\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,p,\alpha}g(z)}-p)\} \\ &\prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}{z^p})^{\mu}\{1+\mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}-p)\} \\ &= p(z)+zp'(z) \end{aligned}$$

Thus in view of Lemma 1.4, $q(z)\prec p(z)$ and p is the best subordinant.

Combining Theorem 2.1 and Theorem 2.3 we get the following sandwich theorem:

Theorem 2.5. *Let $f,g_1,g_2\in\mathcal{A}_p$ and let $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_1(z))^{\Omega}$, $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2(z))^{\Omega}$ be convex univalent functions in U satisfy*

$$\Re\{1+\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2'(z)}+(\Omega-1)\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2(z)}+\frac{1}{\gamma}\}>0.$$

If

$$(i)\quad (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}\in\mathcal{H}[0,1]\cap Q,$$

$$(ii)\quad (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}] \quad is\ univalent\ in\ U$$

and satisfies the subordination

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_1(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_1'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_1(z)}] \prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z)}]$$

$$\prec (\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2(z))^{\Omega}[1+\Omega\gamma\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2(z)}],$$

where $\Omega>0,\gamma\in\mathbb{C}$ with $\Re\{\gamma\}>0$. Then

$$(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_1(z))^{\Omega}\prec(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}f(z))^{\Omega}\prec(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2(z))^{\Omega},\quad \Omega>1,$$

such that $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_1(z))^{\Omega}$ is the best subordinant and $(\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p}g_2(z))^{\Omega}$ is the best dominant.

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Proof. Simultaneously applying the techniques of the proof of Theorem 2.1 and Theorem 2.3, we obtain the required result.

Combining Theorem 2.2 and Theorem 2.4 we get the following sandwich theorem:

Theorem 2.6. *Let* $f, g_1, g_2 \in \mathcal{A}_p$ *and let* $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1(z)}{z^p})^\mu$ *be convex univalent functions in* U. *Assume that*

$$\rho(z) := \frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g(z)} - p, \quad (z \in U)$$

such that

$$\Re\{\frac{(\rho(z)+p)[\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2''(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2'(z)} - (\rho(z)+p)] + p}{\rho(z)} + \mu\rho(z) + 2\} > 0, \quad (z \in U).$$

and

$$(i) \quad z[(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2(z)}{z^p})^\mu]', z\left[\left(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1(z)}{z^p}\right)^\mu\right]' \textit{ are starlike univalent functions in } U$$

$$(ii) \quad (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)}{z^p})^\mu \left\{1 + \mu\left(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)} - p\right)\right\} \textit{ is univalent in } U \textit{ and}$$

$$(iii) \quad (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)}{z^p})^\mu \in \mathcal{H}[0,1] \cap Q.$$

If $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)}{z^p})^\mu \in \mathcal{A}_p$ and the subordination

$$(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1(z)}{z^p})^\mu\{1 + \mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1(z)} - p)\} \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)}{z^p})^\mu\{1 + \mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)} - p)\}$$
$$\prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2(z)}{z^p})^\mu\{1 + \mu(\frac{z\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2'(z)}{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2(z)} - p)\}$$

holds then

$$(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1(z)}{z^p})^\mu \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} f(z)}{z^p})^\mu \prec (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2(z)}{z^p})^\mu, \quad \mu \geq 1,$$

and $(\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_1(z)}{z^p})^\mu, (\frac{\mathcal{D}^{k,\alpha}_{\delta,\beta,\lambda,p} g_2(z)}{z^p})^\mu$ are respectively the best dominant and the best subordinant.

Proof. By using the same techniques, as in the proof of Theorem 2.2 and Theorem 2.4, the required result is obtained.

III. CONCLUSION

we study a new subclass of p-valent function by using the subordination concept between this function and a generalised derivative operator in the open unit disc.

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