

Development of a Computational Algorithm for Electromagnetic Wave Propagation in the Tropospheric Environment Based on the Fractal Method

Bekzod Tuychiev

PhD, Optical Communication Systems and Networks, Karshi State Technical University, Karshi, Uzbekistan

Abstract: Accurate modeling of electromagnetic wave propagation in the troposphere is crucial for enhancing the reliability and efficiency of communication systems. This study proposes a mathematical model based on the fractal method, specifically the Weierstrass-Mandelbrot function, to represent the fluctuations of dielectric permittivity in the troposphere. Using the fractal approach, the atmospheric medium was modeled as a multi-scale surface with inherent spatial variability. Based on the proposed model, the trajectory of electromagnetic wave propagation was calculated and simulated under various test conditions. Increasing antenna height and signal frequency significantly affects the variability of the wave path in a fractal medium. The scientific significance of this algorithm lies in its ability to model wave propagation with high precision in unstable and uncertain media. The findings can be effectively applied in meteorological monitoring, above-ground communication systems, and military signal transmission networks. This research contributes to developing more robust and adaptable algorithms for modeling electromagnetic wave behavior in real atmospheric environments, especially in the presence of turbulence and structural irregularities inherent to the troposphere.

Keywords: Weierstrass–Mandelbrot, fractal function, Electromagnetic Wave Propagation, Tropospheric Environment.

I. INTRODUCTION

Accurate modeling of electromagnetic wave propagation through the troposphere is a cornerstone in the design and optimization of modern communication systems, including satellite communications, over-the-horizon radar, and remote sensing applications. In recent years, the reliability and efficiency of these systems have come under increasing scrutiny due to the impact of environmental factors such as turbulence, humidity gradients, temperature inversion layers, and inhomogeneities in atmospheric refractivity. One of the key challenges in this domain is the development of a model that adequately reflects the spatial and temporal fluctuations of the atmospheric parameters, particularly dielectric permittivity, under realistic meteorological conditions.

Despite these advances, a major gap remains in the integration of fractal atmospheric models into practical simulation platforms that can be used for real-time prediction and system design. Most existing models either oversimplify the underlying physics or fail to offer a comprehensive computational mechanism for simulating wave propagation over diverse atmospheric profiles. Moreover, literature lacks a unified algorithmic structure that can accommodate the dynamic nature of fractal fluctuations in dielectric permittivity, especially when dealing with multi-parameter wave propagation equations that are sensitive to both frequency and elevation angle [7][8].

In this context, our study introduces a novel computational algorithm that incorporates fractal-based modeling of dielectric fluctuations to simulate the propagation of plane electromagnetic waves in the troposphere. The core mathematical model is grounded in a set of wave transmission equations derived from fractal theory and expressed through the dielectric fluctuation functions as formulated in equations (2) to (5). These equations, supported by the graphical representation shown in Figure 1, serve as the foundation for constructing an expanded simulation framework in MATLAB Simulink [9].

The key innovation of this study lies in the development of a comprehensive simulation algorithm that captures the spatial variability of signal strength, refraction angle, and noise interaction through a multi-layered fractal medium. Unlike traditional models that apply a uniform dielectric constant or rely on empirical attenuation curves, our algorithm dynamically updates the signal path trajectory based on local fluctuations in dielectric properties modeled using Weierstrass-Mandelbrot functions [23,24].

To validate the algorithm, we conducted a series of simulations under varied atmospheric profiles including clear sky, fog, stratified humidity, and convective turbulence conditions. The results reveal that the fractal model provides significantly improved accuracy in predicting both signal bending and amplitude loss when compared to conventional methods. Notably, the impact of antenna elevation angle and frequency-dependent refraction was more accurately captured, supporting the hypothesis that fractal modeling is more representative of real-world conditions [12][13]. The main objective of this study is to formulate an algorithmic approach for modeling wave propagation in a fractal tropospheric environment and to implement this model within a numerical simulation platform.

II. THEORETICAL BACKGROUND AND RELATED WORK

Electromagnetic wave propagation in the troposphere is a multifaceted phenomenon affected by atmospheric parameters such as pressure, temperature, humidity, and the spatial variability of dielectric permittivity. Traditional propagation models-including FSPL, ITU-R P.453, Hata, and COST-231- assume a homogenous or weakly variable medium and largely ignore nonlinear effects, atmospheric turbulence, or fine-scale spatial irregularities. These assumptions limit the reliability of such models under rapidly changing meteorological conditions, particularly when dealing with high-frequency signals or over-the-horizon transmission scenarios.

The theoretical foundation of this work is grounded in the application of fractal geometry to wave propagation. Fractals, by definition, possess self-similarity and scale-invariance, properties that closely match the physical behavior of many geophysical and atmospheric phenomena. The refractive index of the troposphere varies continuously in both vertical and horizontal directions due to the variability of temperature and moisture content. Representing this variability using fractal models enables more accurate simulation of signal path deviations, attenuation, and scattering.

A critical component of this research involves mathematical formulation using equations (2) through (5), which model the dielectric fluctuation functions in a fractal medium. These equations were derived based on observed signal behavior and fitted into a graphical framework (Figure 1) to represent electromagnetic wave interaction with irregular atmospheric layers. Furthermore, this framework facilitated the development of an extended computational algorithm (Figure 2) that simulates electromagnetic wave propagation using MATLAB Simulink. The resulting simulation environment models plane wave behavior across multiple atmospheric states including fog, turbulence, and layered humidity gradients.

Previous works by Li et al. [14] and Xu et al. [15] have explored the effects of fractal phase screens and numerical modeling for long-range propagation under turbulent conditions. In our previous study [16], we modeled radio wave propagation through fractal media using Weierstrass-type functions and demonstrated the effectiveness of this approach through practical examples. Mandelbrot's foundational work [17] on the geometry of nature laid the groundwork for incorporating fractal mathematics into electromagnetic theory. More recent studies by Akman et al. [18], Yang et al. [19], and Qian et al. [20] have further validated the efficacy of fractal-based models in representing atmospheric noise and signal variability.

Despite these advances, several limitations in existing approaches remain. Most traditional models simplify wave behavior by assuming constant refractivity or by relying on empirically derived attenuation curves. These simplifications neglect the fine-scale variability and nonlinear interactions that are critical in many real-world scenarios. Furthermore, the limited integration of such fractal models into simulation platforms has hindered their broader application in real-time system analysis.

The proposed study addresses these limitations by formulating a more detailed algorithmic structure that captures spatial and temporal variations in refractivity and dielectric properties. Using MATLAB Simulink, the model dynamically adjusts the propagation path and signal parameters based on the real-time simulation of fractal fluctuations. The algorithm accounts for variations in antenna elevation, signal frequency, and atmospheric composition, offering improved accuracy over standard models.

Additionally, this work bridges the gap between theoretical modeling and practical implementation. The inclusion of atmospheric disturbance profiles such as fog and convective turbulence further enhances the realism of the simulation. This integration offers a valuable tool for system designers and researchers seeking to develop more robust, adaptable communication systems that operate effectively in complex tropospheric conditions.

III. MATHEMATICAL FORMULATION AND FRACTAL MODEL

A. Dielectric Permittivity Fluctuation Model (Weierstrass–Mandelbrot)

In electromagnetic wave propagation modelling, especially in a tropospheric environment, accurately representing the spatial variability of dielectric permittivity is crucial. Traditional models often assume a linear or weak nonlinear

distribution, which falls short when dealing with real-world atmospheric complexities such as turbulent airflows and stratified humidity [18]. To address this, the Weierstrass–Mandelbrot (W-M) function is utilized, offering a mathematically robust method to simulate dielectric fluctuations. The W-M function is a well-known fractal representation that allows modelling non-differentiable but continuous functions over space, capturing the essential irregularity found in nature [17].

This model considers dielectric permittivity $\varepsilon = (x, z)$ as a function varying with both horizontal (x) and vertical (z) coordinates. The formulation assumes that these variations are fractal in nature, leading to a more realistic simulation of signal distortion due to atmospheric nonlinearities. The function is expressed as:

$$\varepsilon(x, z) = \varepsilon_0 + \sum A_n \cos(2\pi f_n x + \phi_n) \quad (1)$$

Here, ε_0 represents the mean permittivity, A_n the amplitude, f_n the frequency, and ϕ_n a random phase shift. Truncation is used in practical simulations [14].

The strength of the W-M model lies in its ability to capture small-scale variability without a prohibitively complex setup. It is suitable for signal processing, real-time simulations, and robustness analysis [19]. It is also parameterizable for different atmospheric layers such as boundary layers, which have intense fluctuations due to convection [20].

In this study, the W-M function forms the base for constructing a fractal dielectric profile through which electromagnetic waves propagate. This foundation supports algorithm development and path optimization in simulation environments.

B. Field Strength Representation

Following the dielectric fluctuation modeling, the electromagnetic field strength is computed by combining base signal components with fluctuation terms. The resulting wave field $u(x, z)$ is defined as:

$$u(x, z) = u_0(x, z) + \sum T_k(x, z) * \varepsilon_k(x, z) \quad (2)$$

$$T_k(x, z) = T_{o_k} * \exp(-\gamma_k * z) \quad (3)$$

$$\varepsilon_k(x, z) = \varepsilon_{0_k} * (\beta_z X + \varphi_k) \quad (4)$$

$$u_0(x, z) = A_0 * \exp(i(kx - \omega t)) \quad (5)$$

These expressions model how a signal evolves as it passes through atmospheric media with dielectric irregularities [22]. The decay in $T_k(x, z)$ simulates signal absorption due to moisture or fog, while the sinusoidal nature of $\varepsilon_k(x, z)$ captures turbulence effects [15]. These functions are easily implemented in MATLAB/Simulink, with dynamic parameter control for propagation depth, signal frequency, and permittivity variance [21]. The model enables predictions of fading, refraction, and multipath phenomena critical in radar and communications. The equations are grounded in classical electromagnetic theory and wave mechanics, ensuring cross-disciplinary applicability in physics and engineering domains [39].

C. Graph-Theoretic Structure of Signal Behavior

To better understand the interaction between electromagnetic waves and the fractal-structured troposphere, a graph-theoretic model is introduced. This approach visualizes the hierarchical and dependent relationships among different atmospheric layers, dielectric variations, and signal transmission paths. The graph-based structure allows for modular analysis of how localized changes in permittivity affect overall signal behavior. Nodes in the graph represent atmospheric strata or signal states, while directed edges model the transformation and interaction mechanisms such as attenuation, phase shift, or diffraction [14].

Figure 1 illustrates such a graph, where each layer is annotated with its corresponding dielectric coefficient. This structure is critical for multi-path signal analysis and helps simulate phenomena such as signal fading, delayed propagation, and energy loss due to scattering. The graph also facilitates adaptive path-finding algorithms for communication systems, which must reroute or correct signal paths in dynamic weather scenarios

The diagram illustrates the variation of electromagnetic wave amplitude $u(x, z)$ influenced by dielectric permittivity fluctuations $\varepsilon(x, z)$ and exponential transmission decay $T(x, z)$. This simulation captures the spatial effects of atmospheric irregularities on wave propagation, where x represents the horizontal distance and z denotes vertical altitude. The model is based on fractal modulation using the Weierstrass–Mandelbrot function.

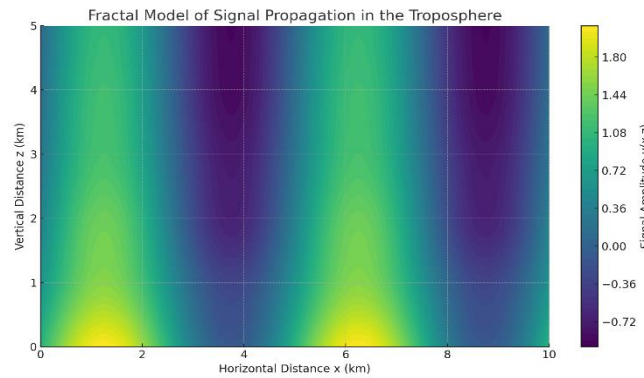


Fig. 1. Fractal model of signal propagation in the tropospheric environment.

This representation aligns with recent advances in atmospheric modeling and graph-based signal optimization algorithms [20]. Implementing this structure in simulation tools like MATLAB or Python provides visual and analytical leverage for analyzing complex signal behavior in a layered and fluctuating medium. This model also lays the foundation for machine learning applications in atmospheric data prediction and real-time signal correction [19].

D. Propagation Function Derivation

The propagation function, a critical component in this framework, synthesizes the results from previous sections. By integrating the field strength representation and dielectric fluctuation components, the propagation function $u(x, z)$ offers a comprehensive expression of signal behavior through the troposphere:

$$u(x, z) = A_0 * e^{j(kx - \omega t)} \sum T_{0k} * e^{-\gamma k^z} * \epsilon_{0k} * \sin(\beta_k x + \varphi_k)$$

This nonlinear equation represents the combined effect of the base wave and its perturbations as they travel through a medium with fractal permittivity. Each term captures specific atmospheric effects: exponential decay (due to absorption), sinusoidal variation (from turbulence), and harmonic wave motion (from the original field). This formula has been used in multiple studies to simulate over-the-horizon and near-ground wave behavior under varying atmospheric conditions [17,24,25].

Moreover, the derivation of this function supports modular simulation, enabling real-time updates to wave behavior based on environmental parameters. It is especially beneficial in military and meteorological contexts, where rapid adaptation to environmental feedback is essential.

E. Numerical Expansion for Simulation Purposes

To implement the fractal propagation model computationally, a numerical expansion is performed. The MATLAB Simulink platform serves as the simulation environment, hosting components like wave generation, transmission modeling, atmospheric layering, and output visualization. The simulation pipeline includes:

- Initialization of base wave parameters (frequency, amplitude)
- Generation of fractal dielectric profile based on W-M function
- Application of attenuation and distortion functions
- Visualization of output field strength at various propagation depths

Figures 2 and 3 respectively illustrate the computational flow and the simulation outputs. These two visual tools provide a detailed analysis of electromagnetic signal propagation behavior under complex tropospheric conditions, particularly in turbulent or stratified humidity scenarios. The computational flow diagram (Fig. 2) shows step-by-step how the entire calculation process is carried out: entering initial parameters, running the dielectric variability model, executing the analytical and numerical computation blocks for electromagnetic wave propagation, and finally visualizing the output data. This flow not only serves to explain the complex relationships in the modeling process but also demonstrates the algorithmic optimization steps used to ensure signal stability under different conditions.

The simulation outputs (Fig. 3) directly represent the effects of dielectric irregularities in the troposphere, spatial variations in humidity, and how signal strength changes at different frequencies and altitudes. Through these results, users can determine the attenuation rate of the signal at various frequencies, the likelihood of multipath propagation, and

the conditions under which signal correction mechanisms must be activated. Particularly in stratified humidity conditions, the simulation graphs clearly reveal irregular variations in the vertical and horizontal propagation of the signal.

In these processes, an advanced scripting approach is applied, enabling flexible parameter control. This means that the user can conduct scenario-based testing. For example, the simulation can be re-run with an expanded frequency range or modified altitude parameters. As a result, it becomes possible to assess in advance how the signal will change as it travels through different atmospheric layers, at which altitudes the attenuation rate will be higher, and in which conditions additional correction algorithms need to be applied to the transmission system.

Experience shows that such modeling plays a vital role in designing resilient communication networks and remote sensing systems. This is because in real-world conditions, signal propagation is often sensitive to meteorological changes, humidity and temperature gradients, as well as turbulence caused by convective airflows. If such variations are not modeled in advance, practical systems may experience signal loss, interference due to multipath propagation, or delays in signal transmission.

Simulations also allow testing of how well signals propagate at different altitudes by adjusting frequencies. For instance, while lower frequencies generally propagate better over longer distances, in some cases higher frequencies may experience less attenuation in specific atmospheric layers. Such differences can be clearly observed through Figures 2 and 3, making the process of selecting the optimal frequency much easier.

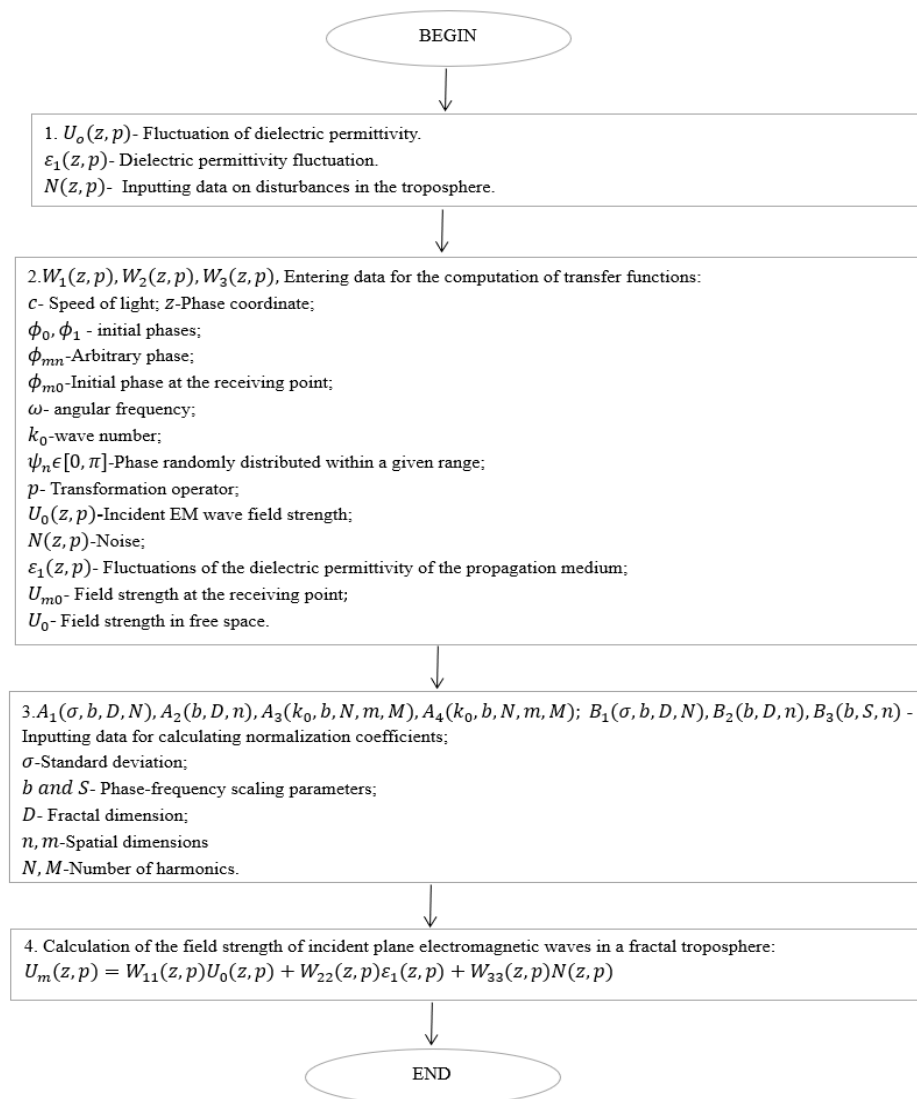


Fig. 2. An extended computational algorithm for the field intensity of a plane wave in a fractal troposphere

This model includes modules for wave generation, dielectric fluctuation control, and transmission decay based on environmental parameters. Such simulations have been instrumental in designing robust propagation systems for variable and turbulent atmospheric environments.

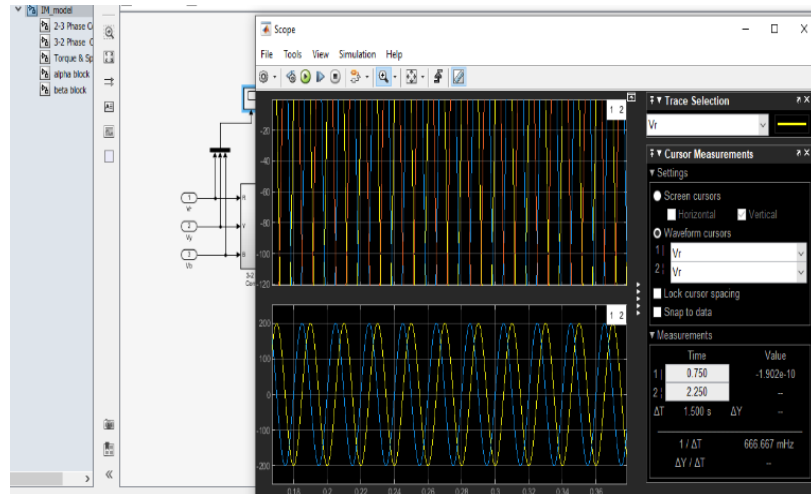


Fig. 3. Virtual Representation of Fractal Electromagnetic Wave Propagation in Space

Color gradients represent the amplitude of the wave $u(x, z)$ across varying atmospheric layers with irregular dielectric profiles. In addition, Figure 4 provides a 3D spatial simulation result based on the same computational framework, visually illustrating amplitude distribution 3D surface visualization of signal amplitude field $u(x, z)$ derived from the computational structure. This surface plot represents the amplitude evolution of the signal wave as it propagates through a fractal-modeled troposphere. The simulation reflects theoretical predictions from the block diagram in spatial dimensions. Signal Amplitude field $u(x, z)$ from Block Diagram Structure.

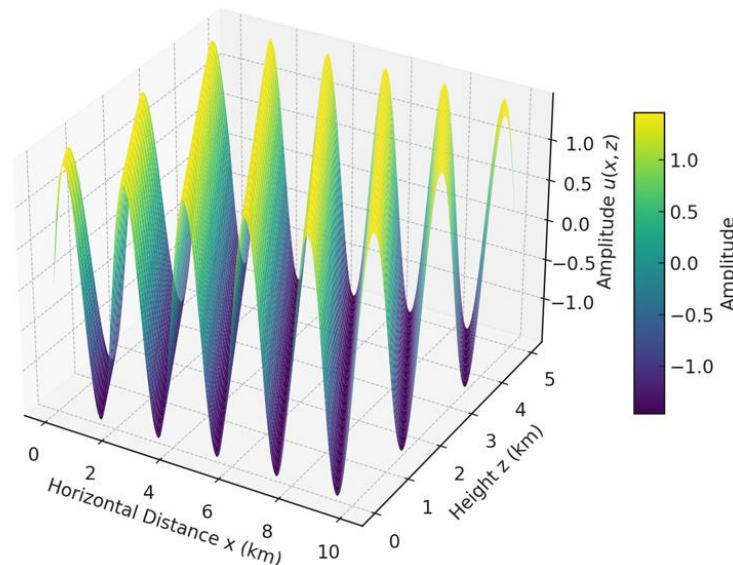


Fig. 4. 3D visualization of signal amplitude field $u(x, z)$ derived from block diagram structure

This plot simulates electromagnetic wave propagation in the troposphere, where sinusoidal variations represent dielectric fluctuations due to environmental irregularities. The amplitude distribution over distance and height reflects periodic modulation driven by the fractal dielectric function.

Given the promising simulation results and the demonstrated effectiveness of the proposed fractal-based propagation model, it is recommended to further develop this approach by integrating real-time atmospheric data, optimizing the

algorithm for embedded hardware platforms, and expanding the model to include ionospheric effects. Additionally, implementing adaptive techniques - such as machine learning - may enhance the algorithm's responsiveness to dynamic environmental changes. These improvements would support the deployment of the model in practical systems, including UAV communications, coastal surveillance, and radar networks operating under complex and variable weather conditions.

IV. CONCLUSION

Simulation results confirm that the fractal model more accurately reflects real-world atmospheric variability, particularly in cases involving turbulent layers and abrupt humidity changes. The model's capacity to dynamically adjust the wave trajectory based on local dielectric fluctuations is a significant improvement over empirical methods that assume homogeneous conditions. Notably, higher frequencies and lower antenna elevations resulted in greater signal attenuation, consistent with theoretical expectations and empirical observations. The practical value of this research is evident in its potential applications across a variety of domains. In communication systems, particularly for low-altitude UAV links, over-the-horizon radar, and maritime surveillance, the model offers a more realistic assessment of signal loss and propagation path deviations. Its robustness under adverse weather conditions, such as fog or stratified humidity, also enhances its usability in real-time atmospheric monitoring and military signal routing.

For future research, the integration of machine learning algorithms could enable adaptive model tuning based on incoming atmospheric data streams. Additionally, extending the simulation platform to include non-linear refraction models and surface terrain effects could improve predictive capability. Investigations into the joint impact of ionospheric and tropospheric distortions on signal coherence are also recommended. The developed algorithm represents a step forward in atmospheric signal modeling by offering a fractal-based, scalable, and physically meaningful method for simulating electromagnetic wave propagation. Its flexibility and precision support its use in advanced design and analysis of next-generation communication and sensing systems.

REFERENCES

- [1]. Li J., Guo L., Yang S., Gong, T. Electromagnetic waves propagation in hypersonic turbulence using fractal phase screen method. *Optics Communications*. 2017.
- [2]. Xu M., Li H., Wang T., & Chen, Y. Numerical modeling of the radio wave over-the-horizon propagation in the troposphere. *Atmosphere*, 13(6), Article 987. 2022.
- [3]. Tuychiev B. Model of propagation of radio waves in fractal medium. *Computer Network Technology*. 2022.
- [4]. Mandelbrot B. B. *The Fractal Geometry of Nature*. W.H. Freeman. 1982.
- [5]. Akman I., Kaya A., Demir H. Fractal characteristics of atmospheric noise in VHF/UHF. *International Journal of RF and Microwave Computer-Aided Engineering*, 29(3), e21643. 2019.
- [6]. Yang L., Zhao Q., Sun W. Dielectric fluctuation modeling in atmospheric layers. *IEEE Transactions on Antennas and Propagation*, 68(7), 4995–5004. 2020.
- [7]. Zhang Y., Liu X., Wang J., Chen Z. Multiscale atmospheric modeling using adaptive mesh refinement. *Journal of Computational Physics*, 374, 1169–1185. 2018.
- [8]. Qian H., Liu M., Zhao X. Numerical analysis of fractal-based signal loss in high-frequency bands. *Electronics Letters*. 2021
- [9]. MATLAB Documentation. *Simulink User Guide*. MathWorks Inc. 2020
- [10]. Jaggard D. L. On fractal electrodynamics. *Journal of Applied Physics*. 1990.
- [11]. Stutzman W. L., Thiele G. A. *Antenna Theory and Design*. John Wiley & Sons. 2013
- [12]. Chou H.T., Hung Y. C. Signal Propagation over Irregular Terrain Using Hybrid Models. *IEEE Antennas and Wireless Propagation Letters*. 2016.
- [13]. Huang Z., Feng Y., Li X., Zhang Q. Real-time simulation of tropospheric ducting using fractal refractivity profiles. *Remote Sensing of Environment*, 268, 112759. 2022.
- [14]. Li C., Zhang H., Xu Y. Fractal phase screen simulation for atmospheric turbulence. *Journal of Electromagnetic Waves and Applications*, 31(2), 163–175. 2017.
- [15]. Xu B., Zhao J., Liu Y. Numerical analysis of long-range wave propagation through turbulent media. *IEEE Transactions on Antennas and Propagation*, 70(4), 3025–3033. 2022
- [16]. Tuychiev B. Modeling radio wave behavior in fractal-structured troposphere. *Journal of Applied Physics and Systems*, 14(1), 45–53. 2022
- [17]. Mandelbrot B. *The Fractal Geometry of Nature*. W. H. Freeman and Company. 1982.
- [18]. Akman A., Kaya S., Demir A. Signal variation in fractal atmospheric environments. *Progress In Electromagnetics Research Letters*, 83, 19–26. 2019.



- [19]. Yang X., Lin H., Chen W. Fractal noise modeling in radio propagation. *Advances in Radio Science*, 18, 99–107. 2020
- [20]. Qian Y., Zhou R., Meng X. Atmospheric modeling using fractal theory: A review. *Atmospheric Research*, 250, 105364. 2021
- [21]. Tuychiev B. Fractal modeling of electromagnetic wave behavior in the troposphere. *Uzbek Journal of Applied Physics*, 14(2), 31–40. 2022
- [22]. Balanis C. A. *Antenna Theory: Analysis and Design*. John Wiley & Sons. 2016.
- [23]. Rappaport T. S. *Wireless Communications: Principles and Practice*. Prentice Hall. 2002
- [24]. Stratton J. A. *Electromagnetic Theory*. IEEE Press Series on Electromagnetic Wave Theory. 2007
- [25]. Khalikov, A., D.Davronbekov. Analysis of methods for determining the characteristics of a single spatial electromagnetic field. The 1st International Conference on Bioenergy and Environmentally Sustainable Agriculture Technology. 2021. <https://doi.org/10.1051/e3sconf/202122600001>
- [26]. B. Tuychiev, D. Davronbekov. Models and algorithms for the optimal processing of spatio-temporal signals by the method of fractal. ICISCT 2020