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SOME RESULTS ON ROUGH NEUTROSOPHIC IDEALS OF BCK-ALGEBRAS

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Abstract: The aim of this paper is to apply the principle of Rough neutrosophic ideals to a BCK-algebra. We establish structural characteristics of each and several related properties, theorems, and are investigated in this paper.

Keywords: Rough neutrosophic set, Neutrosophic sub - algebra, Rough neutrosophic ideals.

I. INTRODUCTION

Many dynamic systems and many functional environments, such as behavioural, biologial, and chemical, face various forms of uncertainties. Zadeh [17] introduced the fuzzy set in 1965 to deal with uncertainties in a variety of real-world applications. Pawlak [15] (1982, 1991) introduced the rough set as a methodology for dealing with imperfect information, especially vague concepts. Many scholars and researchers [2][14] from around the world are focused in rough set theory. The rough set approach is essential in artificial intelligence and cognitive sciences, specifically in learning algorithms, data mining, and information retrieval.

As a generalisation of the fuzzy set, Atanassov [1] proposed the intuitionistic fuzzy set on a universe in 1983. Smarandache [3][4] [5] develops the notion of neutrosophic set as a more specific platform that develops the notions of classic set, intuitionistic fuzzy set, and interval valued intuitionistic fuzzy set.

The ideas of BCK-algebras and BCI-algebras were established by Imai and Iséki [6][7] in 1966 to characterise BCK/BCI-logics, which are components of the propositional calculus containing implication. The class of BCK-algebras is considered to be a proper subclass of the class of BCI-algebras. In BCK-algebra, Y. B. Jun (along with Hong, Kim, Meng, Roh, et.al) [8][9][10][11] studied the fuzzification of ideals and sub-algebras.

In this article, we establish the idea of sub-algebras and ideals in BCK - algebras as a rough neutrosophic and study some of their properties.

II. PRELIMINARIES

Let $\Im$ denote the BCK-algebra throughout this session and C_R denote the congruence relation on $\Im$.

Definition 2.1 [12]: An algebra $(X,*,0)$ of type (2,0) is called a BCK-algebra if it satisfies the following axioms:

(i) $((x*y)*(x*z))*(z*y)=0$,

(ii) $(x*(x*y))*y=0$,

(iii) $x*x=0$,

(iv) $x*y=0$ and $y*x=0$ imply that $x=y$

(v) $0*x=0$ for all $x,y,z\in X$.

Definition 2.2 [12]: A partial ordering $"\leq"$ on X can be defined by $x\leq y$ if and only if $x*y=0$.

Definition 2.3 [12]: In any BCK-algebra X the following holds:

(i) $x*0=x$

(ii) $x*y\leq x$

(iii) $(x*y)*z=(x*z)*y$

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(iv) $(x*z)*(y*z) \le x*y$

(v) $x*(x*(x*y)) = x*y$

(vi) $x \le y \Rightarrow x*z \le y*z$ and $z*y \le z*x$, for all $x, y, z \in X$.

Definition 2.4 [12]: A subset S of a BCK-algebra X is called a subalgebra of X if $x*y \in S$ whenever $x, y \in S$.
Definition 2.5 [12]: A non-empty subset I of a BCK-algebra X is called an ideal of X if it satisfies:
(i) $0 \in I$,
(ii) $x*y \in I$ and $y \in I$ imply $x \in I$

Proposition 3.6 [12]: In a BCK-algebra X, the following holds, for all $x, y, z \in X$.

$$(i)((x*z)*z)*(y*z) \le (x*y)*z,$$

$$(ii)(x*z)*(x*(x*z)) = (x*z)*z,$$

$$(iii)(x*(y*(y*x)))*(y*(x*(y*(y*x)))) \le x*y.$$

III. ROUGH NEUTROSOPHIC IDEAL OF BCK-ALGEBRA.

In this session Rough neutrosophic sub algebra (RNSA), Rough neutrosophic ideal(RNI) of BCK-algebra($\Im$) is introduced and related theorems and properties were discussed.
Definition 3.1: A Neutrosophic sub algebra (NSA) is said to be Rough neutrosophic sub algebra (RNSA) of $\Im$ if it is both lower RNSA and upper RNSA of $\Im$.
Definition 3.2: A NSA is said to be lower(upper) RNSA of $\Im$ if its lower(upper) approximation is also an RNSA of $\Im$.
Proposition 3.3: Every $\underline{C_R}$ lower NSA of $\Im$ satisfies,

$$\underline{C_R}(\kappa_t)(0) \ge \underline{C_R}(\kappa_t)(\alpha); \underline{C_R}(\kappa_i)(0) \le \underline{C_R}(\kappa_i)(\alpha); \underline{C_R}(\kappa_f)(0) \le \underline{C_R}(\kappa_f)(\alpha)$$
$\forall \alpha \in \Im$.

Proof: We have for any $f \in \Im$,

$$\begin{aligned}
\underline{C_R}(\kappa_t)(0) &= \wedge_{g*g \in [f*f]_\Im} \underline{C_R}(\kappa_t)(f*f) \\
&\ge \wedge_{g*g \in [f*f]_\Im} min\{\underline{C_R}(\kappa_t(f)), \underline{C_R}(\kappa_t(f))\} \\
&= \wedge_{g \in [f]_\Im, g \in [f]_\Im} min\{\underline{C_R}(\kappa_t(f)), \underline{C_R}(\kappa_t(f))\} \\
&= min\{\wedge_{g \in [f]_\Im} \underline{C_R}(\kappa_t(f)), \wedge_{g \in [f]_\Im} \underline{C_R}(\kappa_t(f))\} \\
&= \underline{C_R}(\kappa_t(f))
\end{aligned}$$

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$$\begin{aligned}
\underline{C_R}(\kappa_i)(0) &= \vee_{g*g\in[f*f]_{\Im}} \underline{C_R}(\kappa_i)(f*f) \\
&\leq \vee_{g*g\in[f*f]_{\Im}} max\{\underline{C_R}(\kappa_i(f)),\underline{C_R}(\kappa_i(f))\} \\
&= \vee_{g\in[f]_{\Im},g\in[f]_{\Im}} max\{\underline{C_R}(\kappa_i(f)),\underline{C_R}(\kappa_i(f))\} \\
&= max\{\vee_{g\in[f]_{\Im}} \underline{C_R}(\kappa_i(f)), \vee_{g\in[f]_{\Im}} \underline{C_R}(\kappa_i(f))\} \\
&= \underline{C_R}(\kappa_i(f))
\end{aligned}$$

$$\begin{aligned}
\underline{C_R}(\kappa_f)(0) &= \vee_{g*g\in[f*f]_{\Im}} \underline{C_R}(\kappa_f)(f*f) \\
&\leq \vee_{g*g\in[f*f]_{\Im}} max\{\underline{C_R}(\kappa_f(f)),\underline{C_R}(\kappa_f(f))\} \\
&= \vee_{g\in[f]_{\Im},g\in[f]_{\Im}} max\{\underline{C_R}(\kappa_f(f)),\underline{C_R}(\kappa_f(f))\} \\
&= max\{\vee_{g\in[f]_{\Im}} \underline{C_R}(\kappa_f(f)), \vee_{g\in[f]_{\Im}} \underline{C_R}(\kappa_f(f))\} \\
&= \underline{C_R}(\kappa_f(f))
\end{aligned}$$

Hence proved.

Proposition 3.4:
Every C_R upper NSA of $\Im$ satisfies,

$$\overline{C_R}(\kappa_t)(0) \leq \overline{C_R}(\kappa_t)(f); \overline{C_R}(\kappa_i)(0) \geq \overline{C_R}(\kappa_i)(f); \overline{C_R}(\kappa_f)(0) \geq \overline{C_R}(\kappa_f)(f)$$
$\forall f \in \Im.$

Proof: Proof: We have for any $f \in \Im$,

$$\begin{aligned}
\overline{C_R}(\kappa_t)(0) &= \vee_{g*g\in[f*f]} \overline{C_R}(\kappa_t)(f*f) \\
&\geq \vee_{g*g\in[f*f]_{\xi}} max\{\overline{C_R}(\kappa_t(f)),\overline{C_R}(\kappa_t(f))\} \\
&= \vee_{g\in[f]_{\xi},g\in[f]_{\xi}} max\{\overline{C_R}(\kappa_t(f)),\overline{C_R}(\kappa_t(f))\} \\
&= max\{\vee_{g\in[f]_{\xi}} \overline{C_R}(\kappa_t(f)), \vee_{g\in[f]_{\xi}} \overline{C_R}(\kappa_t(f))\} \\
&= \overline{C_R}(\kappa_t(f))
\end{aligned}$$

$$\begin{aligned}
\overline{C_R}(\kappa_i)(0) &= \wedge_{g*g\in[f*f]_{\xi}} \overline{C_R}(\kappa_i)(f*f) \\
&\geq \wedge_{g*g\in[f*f]_{\xi}} min\{\overline{C_R}(\kappa_i(f)),\overline{C_R}(\kappa_i(f))\} \\
&= \wedge_{g\in[f]_{\xi},g\in[f]_{\xi}} min\{\overline{C_R}(\kappa_i(f)),\overline{C_R}(\kappa_i(f))\} \\
&= min\{\wedge_{g\in[f]_{\xi}} \overline{C_R}(\kappa_i(f)), \wedge_{g\in[f]_{\xi}} \overline{C_R}(\kappa_i(f))\} \\
&= \overline{C_R}(\kappa_i(f))
\end{aligned}$$

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$$\begin{aligned}
\overline{C_R}\left(\kappa_f\right)(0) &= \wedge_{g*g\in[f*f]_\xi} \overline{C_R}\left(\kappa_f\right)(f*f) \\
&\le \wedge_{g*g\in[f*f]_\xi} min\{\overline{C_R}(\kappa_f(f)),\overline{C_R}(\kappa_f(f))\} \\
&= \wedge_{g\in[f]_\xi, g\in[f]_\xi} min\{\overline{C_R}(\kappa_f(f)),\overline{C_R}(\kappa_f(f))\} \\
&= min\{\wedge_{g\in[f]_\xi} \overline{C_R}(\kappa_f(f)), \wedge_{g\in[f]_\xi} \overline{C_R}(\kappa_f(f))\} \\
&= \overline{C_R}(\kappa_f(f))
\end{aligned}$$

Hence proved.

Definition 3.5: A Neutrosophic ideal (NI) is said to be Rough neutrosophic ideal (RNI) of $\Im$ if it is both lower NI and upper NI of $\Im$.

Definition 3.6: A NI is said to be lower (upper) RNI of $\Im$ if its lower (upper) approximation is also an NI of $\Im$.

Lemma 3.7: Let κ be a C_R lower RNI of $\Im$. If $f*g\le h$, holds on $\Im$ then,

$$\begin{aligned}
&\forall f,g,h\in\Im \\
&\underline{C_R}\left(\kappa_t\right)(f)\ge min\{\underline{C_R}\left(\kappa_t(g)\right),\underline{C_R}\left(\kappa_t(h)\right)\} \\
&\underline{C_R}\left(\kappa_i\right)(f)\le max\{\underline{C_R}\left(\kappa_i(g)\right),\underline{C_R}\left(\kappa_i(h)\right)\} \\
&\underline{C_R}\left(\kappa_f\right)(f)\le max\{\underline{C_R}\left(\kappa_f(g)\right),\underline{C_R}\left(\kappa_f(h)\right)\}
\end{aligned}$$

Proof:

Given $f*g\le h.$ Then $(f*g)*h=0, \forall f,g,h\in\Im.$

$$\begin{aligned}
\underline{C_R}\left(\kappa_t\right)(f) &= \wedge_{q\in[f]_\Im} \underline{C_R}\left(\kappa_t\right)(q) = \wedge_{p*q\in[f]_\Im*[h]_\Im, q\in[f]_\Im} min\{\underline{C_R}\left(\kappa_t(p*q)\right),\underline{C_R}\left(\kappa_t(q)\right)\} \\
&\ge \wedge_{(p*q)*r\in([f]_\Im*[h]_\Im)*[g]_\Im, r\in[g]_\Im, q\in[h]_\Im} min\{min\{\underline{C_R}\left(\kappa_t((p*q)*r)\right),\underline{C_R}\left(\kappa_t(r)\right)\},\underline{C_R}\left(\kappa_t(q)\right)\} \\
&= \wedge_{r\in[g]_\Im, q\in[h]_\Im} \{min\{\underline{C_R}\left(\kappa_t(0)\right),\underline{C_R}\left(\kappa_t(r)\right)\},\underline{C_R}\left(\kappa_t(q)\right)\} \\
&= min\{\wedge_{q\in[h]_\Im)} \underline{C_R}\left(\kappa_t(r)\right)\}, \wedge_{r\in[g]_\Im} \underline{C_R}\left(\kappa_t(q)\right)\} = min\{\underline{C_R}\left(\kappa_t(g)\right)\},\underline{C_R}\left(\kappa_t(h)\right)\}
\end{aligned}$$

$$\begin{aligned}
\underline{C_R}\left(\kappa_i\right)(f) &= \vee_{q\in[f]_\Im} \underline{C_R}\left(\kappa_i\right)(q) = \vee_{p*q\in[f]_\Im*[h]_\Im, q\in[f]_\Im} max\{\underline{C_R}\left(\kappa_i(p*q)\right),\underline{C_R}\left(\kappa_i(q)\right)\} \\
&\le \vee_{(p*q)*r\in([f]_\Im*[h]_\Im)*[g]_\Im, r\in[g]_\Im, q\in[h]_\Im} max\{max\{\underline{C_R}\left(\kappa_i((p*q)*r)\right),\underline{C_R}\left(\kappa_i(r)\right)\},\underline{C_R}\left(\kappa_i(q)\right)\} \\
&= \vee_{r\in[g]_\Im, q\in[h]_\Im} \{max\{\underline{C_R}\left(\kappa_i(0)\right),\underline{C_R}\left(\kappa_i(r)\right)\},\underline{C_R}\left(\kappa_i(q)\right)\} \\
&= max\{\vee_{q\in[h]_\Im)} \underline{C_R}\left(\kappa_i(r)\right)\}, \vee_{r\in[g]_\Im} \underline{C_R}\left(\kappa_i(q)\right)\} = max\{\underline{C_R}\left(\kappa_i(g)\right)\},\underline{C_R}\left(\kappa_i(h)\right)
\end{aligned}$$

$$\begin{aligned}
\underline{C_R}\left(\kappa_f\right)(f) &= \vee_{q\in[f]_\Im} \underline{C_R}\left(\kappa_f\right)(q) = \vee_{p*q\in[f]_\Im*[h]_\Im, q\in[f]_\Im} max\{\underline{C_R}\left(\kappa_f(p*q)\right),\underline{C_R}\left(\kappa_f(q)\right)\} \\
&\le \vee_{(p*q)*r\in([f]_\Im*[h]_\Im)*[g]_\Im, r\in[g]_\Im, q\in[h]_\Im} max\{max\{\underline{C_R}\left(\kappa_f((p*q)*r)\right),\underline{C_R}\left(\kappa_f(r)\right)\},\underline{C_R}\left(\kappa_f(q)\right)\} \\
&= \vee_{r\in[g]_\Im, q\in[h]_\Im} \{max\{\underline{C_R}\left(\kappa_f(0)\right),\underline{C_R}\left(\kappa_f(r)\right)\},\underline{C_R}\left(\kappa_f(q)\right)\} \\
&= max\{\vee_{q\in[h]_\Im)} \underline{C_R}\left(\kappa_f(r)\right)\}, \vee_{r\in[g]_\Im} \underline{C_R}\left(\kappa_f(q)\right)\} = max\{\underline{C_R}\left(\kappa_f(g)\right)\},\underline{C_R}\left(\kappa_f(h)\right)
\end{aligned}$$

Hence proved.

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Lemma 3.8: Let κ be a C_R upper RNI of $\Im$. If $f*g \le h$, holds on $\Im$ then,

$$\forall f,g,h \in \Im$$
$$\overline{C_R}(\kappa_t)(f) \ge max\{\overline{C_R}(\kappa_t(g)),\overline{C_R}(\kappa_t(h))\}$$
$$\overline{C_R}(\kappa_i)(f) \le min\{\overline{C_R}(\kappa_i(g)),\overline{C_R}(\kappa_i(h))\}$$
$$\overline{C_R}(\kappa_f)(f) \le min\{\overline{C_R}(\kappa_f(g)),\overline{C_R}(\kappa_f(h))\}$$

Proof:

Given $f*g \le h$. Then $(f*g)*h = 0, \forall f,g,h \in \Im$.

$$\overline{C_R}(\kappa_t)(f) = \bigvee_{q\in[f]_\Im} \overline{C_R}(\kappa_t)(q) = \bigvee_{p*q\in[f]_\Im*[h]_\Im, q\in[f]_\Im} max\{\overline{C_R}(\kappa_t(p*q)),\overline{C_R}(\kappa_t(q))\}$$
$$\le \bigvee_{(p*q)*r\in([f]_\Im*[h]_\Im)*[g]_\Im, r\in[g]_\Im, q\in[h]_\Im} max\{max\{\overline{C_R}(\kappa_t((p*q)*r)),\overline{C_R}(\kappa_t(r))\},\overline{C_R}(\kappa_t(q))\}$$
$$= \bigvee_{r\in[g]_\Im, q\in[h]_\Im} \{max\{\overline{C_R}(\kappa_t(0)),\overline{C_R}(\kappa_t(r))\},\overline{C_R}(\kappa_t(q))\}$$
$$= max\{\bigvee_{q\in[h]_\Im)} \overline{C_R}(\kappa_t(r))\}, \bigvee_{r\in[g]_\Im} \overline{C_R}(\kappa_t(q))\} = max\{\overline{C_R}(\kappa_t(g))\},\overline{C_R}(\kappa_t(h))$$
$$\overline{C_R}(\kappa_i)(f) = \bigwedge_{q\in[f]_\Im} \overline{C_R}(\kappa_i)(q) = \bigwedge_{p*q\in[f]_\Im*[h]_\Im, q\in[f]_\Im} min\{\overline{C_R}(\kappa_i(p*q)),\overline{C_R}(\kappa_i(q))\}$$
$$\ge \bigwedge_{(p*q)*r\in([f]_\Im*[h]_\Im)*[g]_\Im, r\in[g]_\Im, q\in[h]_\Im} min\{min\{\overline{C_R}(\kappa_i((p*q)*r)),\overline{C_R}(\kappa_i(r))\},\overline{C_R}(\kappa_i(q))\}$$
$$= \bigwedge_{r\in[g]_\Im, q\in[h]_\Im} \{min\{\overline{C_R}\kappa_i(\kappa_i(0)),\overline{C_R}(\kappa_i(r))\},\overline{C_R}(\kappa_i(q))\}$$
$$= min\{\bigwedge_{q\in[h]_\Im)} \overline{C_R}(\kappa_i(r))\}, \bigwedge_{r\in[g]_\Im} \overline{C_R}(\kappa_i(q))\} = min\{\overline{C_R}(\kappa_i(g))\},\overline{C_R}(\kappa_i(h))\}$$
$$\overline{C_R}(\kappa_f)(f) = \bigwedge_{q\in[f]_\Im} \overline{C_R}(\kappa_f)(q) = \bigwedge_{p*q\in[f]_\Im*[h]_\Im, q\in[f]_\Im} min\{\overline{C_R}(\kappa_f(p*q)),\overline{C_R}(\kappa_f(q))\}$$
$$\ge \bigwedge_{(p*q)*r\in([f]_\Im*[h]_\Im)*[g]_\Im, r\in[g]_\Im, q\in[h]_\Im} min\{min\{\overline{C_R}(\kappa_f((p*q)*r)),\overline{C_R}(\kappa_f(r))\},\overline{C_R}(\kappa_f(q))\}$$
$$= \bigwedge_{r\in[g]_\Im, q\in[h]_\Im} \{min\{\overline{C_R}(\kappa_f(0)),\overline{C_R}(\kappa_f(r))\},\overline{C_R}(\kappa_f(q))\}$$
$$= min\{\bigwedge_{q\in[h]_\Im)} \overline{C_R}(\kappa_f(r))\}, \bigwedge_{r\in[g]_\Im} \overline{C_R}(\kappa_f(q))\} = min\{\overline{C_R}(\kappa_f(g))\},\overline{C_R}(\kappa_f(h))\}$$

Hence proved.

Lemma 3.9: Let κ be a C_R lower RNI of $\Im$. If, $f \le g$ in $\Im$, then

$$\underline{C_R}(\kappa_t)(f) \ge \underline{C_R}(\kappa_t(g)); \underline{C_R}(\kappa_i)(f) \le \underline{C_R}(\kappa_i(g)); \underline{C_R}(\kappa_f)(f) \le \underline{C_R}(\kappa_f(g)).$$

Proof:

Let $f \le g$ in $\Im$, then $f*g = 0$,

$$\underline{C_R}(\kappa_t)(f) \ge \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\underline{C_R}(\kappa_t(p*q)),\underline{C_R}(\kappa_t(q))\} = min\{\underline{C_R}(\kappa_t(0)), \bigwedge_{q\in[g]_\Im} \underline{C_R}(\kappa_t(q))\} = \underline{C_R}(\kappa_t(g))$$
$$\underline{C_R}(\kappa_i)(f) \le \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\underline{C_R}(\kappa_i(p*q)),\underline{C_R}(\kappa_i(g))\} = max\{\underline{C_R}(\kappa_i(0)), \bigvee_{q\in[g]_\Im} \underline{C_R}(\kappa_i(q))\} = \underline{C_R}(\kappa_i(g))$$
$$\underline{C_R}(\kappa_f)(f) \le \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\underline{C_R}(\kappa_f(p*q)),\underline{C_R}(\kappa_f(g))\} = max\{\underline{C_R}(\kappa_f(0)), \bigvee_{q\in[g]_\Im} \underline{C_R}(\kappa_f(q))\} = \underline{C_R}(\kappa_f(g))$$

Hence proved.

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Lemma 3.10: Let κ be a C_R upper RNI of $\Im$. If, $f \le g$ in $\Im$, then

$$\overline{C_R}(\kappa_t)(f) \le \overline{C_R}(\kappa_t(g)); \overline{C_R}(\kappa_i)(f) \ge \overline{C_R}(\kappa_i(g)); \overline{C_R}(\kappa_f)(f) \ge \overline{C_R}(\kappa_f(g)).$$

Proof:
Let $f \le g$ in $\Im$, then $f * g = 0$,

$$\overline{C_R}(\kappa_t)(f) \le \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\overline{C_R}(\kappa_t(p*q)), \overline{C_R}(\kappa_t(q))\} = max\{\overline{C_R}(\kappa_t(0)), \bigvee_{q\in[g]_\Im} \overline{C_R}(\kappa_t(q))\} = \overline{C_R}(\kappa_t(g))$$

$$\overline{C_R}(\kappa_i)(f) \ge \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\overline{C_R}(\kappa_i(p*q)), \overline{C_R}(\kappa_i(g))\} = min\{\overline{C_R}(\kappa_i(0)), \bigwedge_{q\in[g]_\Im} \overline{C_R}(\kappa_i(q))\} = \overline{C_R}(\kappa_i(g))$$

$$\overline{C_R}(\kappa_f)(f) \ge \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\overline{C_R}(\kappa_f(p*q)), \overline{C_R}(\kappa_f(g))\} = min\{\overline{C_R}(\kappa_f(0)), \bigwedge_{q\in[g]_\Im} \overline{C_R}(\kappa_f(q))\} = \overline{C_R}(\kappa_f(g))$$

Hence proved.

Theorem 3.11: Let κ be a C_R lower (upper) RNI of $\Im$, then for any $f, \upsilon_1, \upsilon_2, ...\upsilon_n \in \Im; (...((f * \upsilon_1) * \upsilon_2) * ...) * \upsilon_n = 0$ implies

$$\underline{C_R}(\kappa_t)(f) \ge min\{\underline{C_R}(\kappa_t(\upsilon_1)), \underline{C_R}(\kappa_t(\upsilon_2)), .., \underline{C_R}(\kappa_t(\upsilon_n))\}$$

$$\underline{C_R}(\kappa_i)(f) \le max\{\underline{C_R}(\kappa_i(\upsilon_1)), \underline{C_R}(\kappa_i(\upsilon_2)), .., \underline{C_R}(\kappa_i(\upsilon_n))\}$$

$$\underline{C_R}(\kappa_f)(f) \le max\{\underline{C_R}(\kappa_f(\upsilon_1)), \underline{C_R}(\kappa_f(\upsilon_2)), .., \underline{C_R}(\kappa_f(\upsilon_n))\}$$

$$\overline{C_R}(\kappa_t)(f) \le max\{\overline{C_R}(\kappa_t(\upsilon_1)), \overline{C_R}(\kappa_t(\upsilon_2)), ..., \overline{C_R}(\kappa_t(\upsilon_n))\}$$

$$\overline{C_R}(\kappa_i)(f) \ge min\{\overline{C_R}(\kappa_i(\upsilon_1)), \overline{C_R}(\kappa_i(\upsilon_2)), ..., \overline{C_R}(\kappa_i(\upsilon_n))\}$$

$$\overline{C_R}(\kappa_f)(f) \ge min\{\overline{C_R}(\kappa_f(\upsilon_1)), \overline{C_R}(\kappa_f(\upsilon_2)), ..., \overline{C_R}(\kappa_f(\upsilon_n))\}$$

Proof:

This proof is obvious from lemma 3.7, lemma 3.8, lemma 3.9 and lemma 3.10 by using induction on n.

Theorem 3.12: Every C_R lower RNI of $\Im$ is a C_R lower RNSA of $\Im$.

Proof:
W.K.T

$$\underline{C_R}(\kappa_t)(f * g) \ge \underline{C_R}(\kappa_t(g)); \underline{C_R}(\kappa_i)(f * g) \le \underline{C_R}(\kappa_i(g)); \underline{C_R}(\kappa_f)(f * g) \le \underline{C_R}(\kappa_f(g)).$$

$$\underline{C_R}(\kappa_t)(f * g) \ge \underline{C_R}(\kappa_t(g)) \ge \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\underline{C_R}(\kappa_t(p*q)), \underline{C_R}(\kappa_t(q))\}$$

$$\ge min\{\bigwedge_{p\in[f]_\Im} \underline{C_R}(\kappa_t(p)), \bigwedge_{q\in[g]_\Im} \underline{C_R}(\kappa_t(q))\} \ge min\{\underline{C_R}(\kappa_t(f)), \underline{C_R}(\kappa_t(g))\}$$

$$\underline{C_R}(\kappa_i)(f * g) \ge \underline{C_R}(\kappa_i(g)) \le \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\underline{C_R}(\kappa_i(p*q)), \underline{C_R}(\kappa_i(q))\}$$

$$\le max\{\bigvee_{p\in[f]_\Im} \underline{C_R}(\kappa_i(p)), \bigvee_{q\in[g]_\Im} \underline{C_R}(\kappa_i(q))\} \le max\{\underline{C_R}(\kappa_i(f)), \underline{C_R}(\kappa_i(g))\}$$

$$\underline{C_R}(\kappa_f)(f * g) \le \underline{C_R}(\kappa_f(g)) \le \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\underline{C_R}(\kappa_f(p*q)), \underline{C_R}(\kappa_f(q))\}$$

$$\le max\{\bigvee_{p\in[f]_\Im} \underline{C_R}(\kappa_f(p)), \bigvee_{q\in[g]_\Im} \underline{C_R}(\kappa_f(q))\} \le max\{\underline{C_R}(\kappa_f(f)), \underline{C_R}(\kappa_f(g))\}$$

Hence proved.

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Theorem 3.13: Every C_R upper RNI of $\Im$ is a C_R upper RNSA of $\Im$.

Proof:

$$\overline{C_R}(\kappa_t)(f*g) \le \overline{C_R}(\kappa_t(g)); \overline{C_R}(\kappa_i)(f*g) \ge \overline{C_R}(\kappa_i(g)); \overline{C_R}(\kappa_f)(f*g) \ge \overline{C_R}(\kappa_f(g)).$$

$$\begin{aligned}
\overline{C_R}(\kappa_t)(f*g) \ge \overline{C_R}(\kappa_t(g)) &\le \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\overline{C_R}(\kappa_t(p*q)), \overline{C_R}(\kappa_t(q))\} \\
&\le max\{\bigvee_{p\in[f]_\Im} \overline{C_R}(\kappa_t(p)), \bigvee_{q\in[g]_\Im} \overline{C_R}(\kappa_t(q))\} \le max\{\overline{C_R}(\kappa_t(f)), \overline{C_R}(\kappa_t(g))\} \\
\overline{C_R}(\kappa_i)(f*g) \ge \overline{C_R}(\kappa_i(g)) &\ge \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\overline{C_R}(\kappa_i(p*q)), \overline{C_R}(\kappa_i(q))\} \\
&\ge min\{\bigwedge_{p\in[f]_\Im} \overline{C_R}(\kappa_i(p)), \bigwedge_{q\in[g]_\Im} \overline{C_R}(\kappa_i(q))\} \ge min\{\overline{C_R}(\kappa_i(f)), \overline{C_R}(\kappa_i(g))\} \\
\overline{C_R}(\kappa_f)(f*g) \le \overline{C_R}(\kappa_f(g)) &\ge \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\overline{C_R}(\kappa_f(p*q)), \overline{C_R}(\kappa_f(q))\} \\
&\ge min\{\bigwedge_{p\in[f]_\Im} \overline{C_R}(\kappa_f(p)), \bigwedge_{q\in[g]_\Im} \overline{C_R}(\kappa_f(q))\} \ge min\{\overline{C_R}(\kappa_f(f)), \overline{C_R}(\kappa_f(g))\}
\end{aligned}$$

Hence proved.

Theorem 3.14: Let κ be a C_R RNSA of $\Im$ then κ is a C_R lower (upper) RNI of $\Im$.

Proof:
By Theorem 3.12 and Theorem 3.13 the proof is obvious.

Theorem 3.15: Let κ be a RNSA of $\Im$ such that $\forall f, g, h \in \Im, f*g \le h, and$

$$\underline{C_R}(\kappa_t)(f) \ge min\{\underline{C_R}(\kappa_t(g)), \underline{C_R}(\kappa_t(h))\}$$
$$\underline{C_R}(\kappa_i)(f) \le max\{\underline{C_R}(\kappa_i(g)), \underline{C_R}(\kappa_i(h))\}$$
$$\underline{C_R}(\kappa_f)(f) \le max\{\underline{C_R}(\kappa_f(g)), \underline{C_R}(\kappa_f(h))\}$$
$$\overline{C_R}(\kappa_t)(f) \le min\{\overline{C_R}(\kappa_t(g)), \overline{C_R}(\kappa_t(h))\}$$
$$\overline{C_R}(\kappa_i)(f) \ge max\{\overline{C_R}(\kappa_i(g)), \overline{C_R}(\kappa_i(h))\}$$
$$\overline{C_R}(\kappa_f)(f) \ge max\{\overline{C_R}(\kappa_f(g)), \overline{C_R}(\kappa_f(h))\}$$

Then κ is a RNI of $\Im$.

Proof: $\forall f, g, h \in \Im$

$$\underline{C_R}(\kappa_t)(0) \ge \underline{C_R}(\kappa_t)(f); (\kappa_i)(0) \le \underline{C_R}(\kappa_i)(f); \underline{C_R}(\kappa_f)(0) \le \underline{C_R}(\kappa_f)(f)$$
$$\overline{C_R}(\kappa_t)(0) \le \overline{C_R}(\kappa_t)(f); (\kappa_i)(0) \ge \overline{C_R}(\kappa_i)(f); \overline{C_R}(\kappa_f)(0) \ge \overline{C_R}(\kappa_f)(f)$$

$By\ hypothesis,\ we've\ for, f*(f*g) \le g,$

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$$\underline{C_R}(\kappa_t)(f) \geq \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\underline{C_R}(\kappa_t(p*q)), \underline{C_R}(\kappa_t(q))\}$$

$$\geq \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} \min\{\bigwedge_{(p*q)*r\in[f]_\Im*[g]_\Im*[h]_\Im, r\in[h]_\Im} min\{\underline{C_R}(\kappa_t((p*q)*r), \underline{C_R}(\kappa_t(r))\}, \underline{C_R}(\kappa_t(q))\}$$

$$\geq \min\{min\{\underline{C_R}(\kappa_t(h))\}, \underline{C_R}(\kappa_t(g))\}$$

$$\underline{C_R}(\kappa_i)(f) \leq \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\underline{C_R}(\kappa_i(p*q)), \underline{C_R}(\kappa_i(q))\}$$

$$\leq \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\bigvee_{(p*q)*r\in[f]_\Im*[g]_\Im*[h]_\Im, r\in[h]_\Im} max\{\underline{C_R}(\kappa_i((p*q)*r), \underline{C_R}(\kappa_i(r))\}, \underline{C_R}(\kappa_i(q))\}$$

$$\leq max\{max\{\underline{C_R}(\kappa_i(h))\}, \underline{C_R}(\kappa_i(g))\}$$

$$\underline{C_R}(\kappa_f)(f) \leq \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\underline{C_R}(\kappa_f(p*q)), \underline{C_R}(\kappa_f(q))\}$$

$$\leq \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\bigvee_{(p*q)*r\in[f]_\Im*[g]_\Im*[h]_\Im, r\in[h]_\Im} max\{\underline{C_R}(\kappa_f((p*q)*r), \underline{C_R}(\kappa_f(r))\}, \underline{C_R}(\kappa_f(q))\}$$

$$\leq max\{max\{\underline{C_R}(\kappa_f(h))\}, \underline{C_R}(\kappa_f(g))\}$$

Also,

$$\overline{C_R}(\kappa_t)(f) \leq \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\overline{C_R}(\kappa_t(p*q)), \overline{C_R}(\kappa_t(q))\}$$

$$\leq \bigvee_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} max\{\bigvee_{(p*q)*r\in[f]_\Im*[g]_\Im*[h]_\Im, r\in[h]_\Im} max\{\overline{C_R}(\kappa_t((p*q)*r), \overline{C_R}(\kappa_t(r))\}, \overline{C_R}(\kappa_t(q))\}$$

$$\leq max\{max\{\overline{C_R}(\kappa_t(h))\}, \overline{C_R}(\kappa_t(g))\}$$

$$\overline{C_R}(\kappa_i)(f) \geq \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\overline{C_R}(\kappa_i(p*q)), \overline{C_R}(\kappa_i(q))\}$$

$$\geq \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} \min\{\bigwedge_{(p*q)*r\in[f]_\Im*[g]_\Im*[h]_\Im, r\in[h]_\Im} min\{\overline{C_R}(\kappa_i((p*q)*r), \overline{C_R}(\kappa_i(r))\}, \overline{C_R}(\kappa_i(q))\}$$

$$\geq \min\{min\{\overline{C_R}(\kappa_i(h))\}, \overline{C_R}(\kappa_i(g))\}$$

$$\overline{C_R}(\kappa_f)(f) \geq \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} min\{\overline{C_R}(\kappa_f(p*q)), \overline{C_R}(\kappa_f(q))\}$$

$$\geq \bigwedge_{p*q\in[f]_\Im*[g]_\Im, q\in[g]_\Im} \min\{\bigwedge_{(p*q)*r\in[f]_\Im*[g]_\Im*[h]_\Im, r\in[h]_\Im} min\{\overline{C_R}(\kappa_f((p*q)*r), \overline{C_R}(\kappa_f(r))\}, \overline{C_R}(\kappa_f(q))\}$$

$$\geq \min\{min\{\overline{C_R}(\kappa_f(h))\}, \overline{C_R}(\kappa_f(g))\}$$

Therefore κ is a RNI of $\Im$.

IV. CONCLUSION

In this paper the concept of Rough neutrosophic ideal of BCK-algebra. is defined in this study. Also some of their properties were examined and related proposition and theorems are proved. For further study these can be used to define the concept other extension of BCK-algebra like implicative ideal, commutative ideal etc.

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