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Reliability Analysis and Optimization of Multi-Component Redundant Repairable Systems with Replacement Policies

Anil Singh[1*], Dr. Sharon Moses[2]

Research Scholar, Department of Mathematics, St. John's College, Affiliated to Dr. Bhimrao Ambedkar University, Agra (India)[1]

Professor, Department of Mathematics, St. John's College, Affiliated to Dr. Bhimrao Ambedkar University, Agra (India)[2]

[*]Corresponding Author

Abstract: This paper presents a reliability analysis and optimization framework for multi-component redundant repairable systems with preventive replacement policies. The system is modeled as a k-out-of-N repairable redundant structure in which at least k components must remain operational for successful system performance. A continuous-time Markov chain (CTMC) approach is employed to describe the stochastic behavior of component failures, repairs, and preventive replacements. The Chapman–Kolmogorov differential equations are formulated and solved using matrix analytical and Laplace transform techniques to obtain transient and steady-state solutions. Important reliability measures such as reliability function, availability, Mean Time to System Failure (MTTF), steady-state probabilities, repair rate, replacement rate, and system failure frequency are derived analytically. A threshold-based preventive replacement policy is incorporated to minimize long-run operational cost while improving system reliability and availability. Numerical illustrations and graphical analyses are presented to study the effects of failure rate, repair rate, redundancy level, and replacement threshold on system performance. The results indicate that preventive replacement policies significantly reduce downtime and operational cost compared with corrective maintenance alone. The optimal replacement threshold is obtained by minimizing the expected total cost and maximizing system availability. The proposed model provides an effective decision-making framework for maintenance optimization in complex industrial and engineering repairable systems.

Keywords: Multi-component repairable system, Redundant systems, Reliability analysis, Continuous-time Markov chain, Preventive replacement policy

I. INTRODUCTION

Reliability and maintenance optimization of engineering systems have become increasingly important in modern industrial applications such as manufacturing systems, communication networks, transportation systems, power plants, aerospace structures, and computer-based infrastructures. Most practical systems are composed of multiple interconnected components operating under uncertain environmental and operational conditions. Failure of one or more components may significantly reduce system efficiency and may ultimately lead to complete system breakdown. To improve operational continuity and reduce failure risk, redundant repairable systems are widely employed in reliability engineering. In such systems, standby or redundant components are incorporated so that the system can continue functioning even after partial component failures. However, continuous operation of repairable systems also introduces maintenance challenges due to repeated failures, repair activities, and increasing operational cost. Consequently, effective replacement and maintenance policies are required to balance system reliability, availability, and economic performance. In this paper, a multi-component k-out-of-N redundant repairable system with threshold-based preventive replacement policy is investigated. The stochastic behavior of the system is modeled using continuous-time Markov chain techniques, where component failures, repairs, and preventive replacements are considered simultaneously. Analytical expressions for transient and steady-state reliability measures are derived, and a cost optimization framework is developed to determine the optimal replacement threshold that minimizes long-run operational cost while maximizing system availability. Numerical and graphical analyses are further performed to examine the influence of system parameters on reliability performance and maintenance efficiency. The study demonstrates that preventive replacement policies significantly improve system reliability and provide an effective strategy for minimizing maintenance expenditure in complex repairable systems.

Hashemi *et al.* (2020) investigated optimal maintenance strategies for coherent systems containing multi-type components operating under different reliability characteristics. The authors developed maintenance optimization

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models considering heterogeneous component behavior and analyzed the influence of preventive maintenance on long-term system reliability and operational cost. Their study demonstrated that appropriate maintenance scheduling significantly improves system availability and reduces failure frequency. The work highlighted the importance of integrating reliability analysis with maintenance decision-making in complex repairable systems. Hamdan *et al.* (2021) studied preventive maintenance optimization for repairable weighted k-out-of-n systems. The authors focused on systems where components possess different reliability weights and operational significance. Using reliability-based optimization techniques, they derived maintenance policies that maximize system availability while minimizing maintenance expenditure. Their results indicated that weighted redundancy structures provide improved fault tolerance and that preventive maintenance substantially enhances system performance compared with corrective maintenance alone. Peiravi *et al.* (2022) analyzed redundancy allocation and optimization strategies for k-out-of-n systems using Markov chain modeling and genetic algorithms. The study emphasized the role of redundancy in improving reliability and reducing system downtime. Numerical results showed that optimized redundancy allocation significantly increases Mean Time to System Failure (MTTF) and operational efficiency. The authors also demonstrated that heuristic optimization techniques are highly effective for solving complex reliability optimization problems. Ruiz-Castro and Acal (2022) developed a stochastic framework for modeling multi-state repairable systems with repair-facility vacation policies. The study employed Markovian Arrival Processes and matrix analytical techniques to derive transient and steady-state reliability measures. Their analysis demonstrated the impact of repair-facility interruptions on system performance and maintenance efficiency. The authors concluded that incorporating repair-vacation policies provides a more realistic representation of industrial repairable systems. Safaei *et al.* (2022) proposed a maintenance optimization model for k-out-of-n systems considering operational safety constraints and second-hand component utilization. The authors analyzed preventive replacement decisions under reliability and economic considerations. Their findings revealed that optimized maintenance policies improve operational lifetime and reduce maintenance costs while maintaining acceptable safety levels. The study also highlighted the practical significance of reusing partially functional components in industrial systems. Torrado (2022) investigated optimal component allocation and replacement-time strategies for parallel systems consisting of dependent multi-type components. The research focused on dependency effects among components and their influence on maintenance policies. The results showed that dependency structures significantly affect system reliability and replacement decisions. The author demonstrated that optimal replacement timing can substantially reduce operational cost and improve long-term system survivability. Eryilmaz (2023) studied age-based preventive replacement policies for discrete-time coherent systems composed of identical independent components. The author developed mathematical models to determine optimal replacement intervals based on component aging characteristics. The study revealed that preventive replacement policies effectively reduce sudden system failures and increase operational reliability. The results further showed that system performance can be significantly enhanced through appropriate age-based maintenance scheduling. Ashrafi *et al.* (2024) analyzed preventive maintenance policies for k-out-of-n systems subjected to fatal shocks. The authors considered the influence of catastrophic external events on system reliability and maintenance optimization. Their findings indicated that preventive maintenance reduces the probability of severe system failure and improves operational continuity under uncertain environmental conditions. The study emphasized the importance of incorporating shock effects into reliability modeling of repairable systems. Li *et al.* (2024) investigated a repairable k-out-of-n retrial system with two failure modes and preventive maintenance. The authors developed stochastic reliability models considering retrial behavior, multiple failure mechanisms, and maintenance activities. Analytical expressions for reliability indices and cost measures were derived using probabilistic techniques. Their results demonstrated that preventive maintenance and retrial management significantly improve system availability and reduce long-run operational cost. Qi et al. (2024) proposed a preventive maintenance optimization framework for weighted k-out-of-n:G systems using survival signature techniques. The study incorporated component importance and reliability contribution into maintenance decision-making. Numerical analysis showed that the survival signature approach provides efficient reliability evaluation and improves optimization accuracy for large-scale systems. The authors concluded that optimized preventive maintenance significantly enhances system reliability and maintenance efficiency. Wu and Asadi (2024) developed a preventive maintenance policy and approximation method for analyzing failure processes in multi-component systems. The study focused on simplifying complex stochastic failure mechanisms while preserving reliability accuracy. The authors derived approximate analytical solutions for maintenance optimization and demonstrated that preventive maintenance reduces system downtime and operational risk. Their results highlighted the effectiveness of approximation techniques in reliability assessment of large-scale repairable systems. Niu *et al.* (2025) investigated preventive replacement policies for parallel and series systems with dependent components under deviation cost structures. They analyzed the influence of component dependency and replacement deviation costs on system reliability and economic performance. Their results showed that dependency among components significantly affects optimal replacement decisions. The study concluded that preventive replacement policies based on cost optimization improve long-run operational efficiency and reduce maintenance expenditure. Sheu (2025) studied optimal replacement policies for kkk-out-of-nnn systems containing idle components. The author developed replacement strategies considering standby behavior and

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operational redundancy. The results indicated that idle standby components improve system reliability and delay complete system failure. The study further demonstrated that optimized replacement scheduling reduces total maintenance cost while maximizing system availability. Wang *et al.* (2025) examined joint optimization of cooperative maintenance and inventory control for multiple k-out-of-n:F systems with shared spare parts. The authors integrated reliability analysis with inventory management and maintenance planning. Their results showed that cooperative maintenance and spare-part sharing significantly improve operational efficiency and reduce maintenance cost. The study highlighted the practical importance of integrating reliability engineering with supply-chain and inventory management strategies. Carballo *et al.* (2026) analyzed preventive maintenance strategies for a two-unit priority standby repairable system. The authors developed stochastic reliability models considering standby redundancy, repair processes, and preventive replacement actions. Their analysis demonstrated that optimized preventive maintenance policies increase system availability and reduce the probability of catastrophic failure. The study concluded that standby redundancy combined with preventive replacement provides an effective reliability improvement mechanism for repairable engineering systems.

II. SYSTEM DESCRIPTION AND ASSUMPTIONS

The proposed system consists of N identical repairable components arranged in a redundant configuration. The system operates successfully when at least k components remain functional; therefore, it is modeled as a K-out-of-N repairable redundant system. Each operating component fails independently with constant failure rate λ, while failed components are repaired with repair rate μ. After repair, components are restored to an "as good as new" condition and re-enter service.

To improve reliability and reduce excessive degradation, a replacement policy is incorporated into the system. Preventive replacement is initiated whenever the number of failed components reaches a predetermined threshold level R. The replacement process occurs with rate θ, restoring the system to its fully operational state. The system is considered failed when the number of failed components exceeds the allowable limit $N - k$.

The stochastic behavior of the system is modeled using a continuous-time Markov chain (CTMC). Let $X(t) = i$ denote the number of failed components at time t, where the state space is defined as

$$S = \{0,1,2,\dots,N-k+1\} \tag{1}$$

Here, state i represents i failed components, while state $N-k+1$ corresponds to complete system failure.
The following assumptions are used in the model development:

(i) All components are statistically identical and independent.
(ii) Failure and repair times follow exponential distributions.
(iii) Failure rate and repair rate are constant.
(iv) Repaired components behave like new components.
(v) Preventive replacement occurs at threshold level RRR.
(vi) Switching between active and standby components is perfect.
(vii) Only one transition (failure, repair, or replacement) occurs at a time.
(viii) The system satisfies the Markov property.

Under these assumptions, the model can be analyzed to obtain reliability measures, steady-state probabilities, availability, mean time to system failure, and optimal replacement policies.

III. MATHEMATICAL MODEL

The proposed multi-component repairable system is modeled using a continuous-time Markov chain (CTMC). Consider a k-out-of-N redundant system consisting of NNN identical components, where at least kkk components must operate successfully for proper system functioning. Let $X(t)$ denote the number of failed components at time ttt. The state of the system is therefore represented by

$$X(t) = i, i = 1,1,2,\dots,N-k+1 \tag{2}$$

where state i indicates that exactly i components are failed. The state $N-k+1$ represents complete system failure because fewer than k operational components remain active.

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The failure rate in state iii depends on the number of operational components $(N - i)$. Hence, the transition rate from state i to state $i+1$ is given by $\lambda_i = (N - i)\lambda$ (3)

where λ is the failure rate of each component.
Similarly, the repair rate depends on the number of failed components. Therefore, the transition rate from state i to state $i-1$ is $\mu_i = i\mu$ (4)

where μ is the repair rate of each failed component.
In addition to repair, a preventive replacement policy is considered. Whenever the number of failed components reaches the threshold level R, the system undergoes replacement with replacement rate θ. The replacement action restores the system to the fully operational state 000.

$P_i(t)$ denote the probability that the system is in state iii at time t. The probabilistic behavior of the system is governed by the following Chapman–Kolmogorov differential equations.

For state 0, $\frac{dP_0}{dt} = -N\lambda P_0(t) + \mu P_1(t) + \theta \sum_{j=R}^{N-k} P_j(t)$ (5)

For intermediate states $1 \le i \le N - k$,

$$\frac{dP_i(t)}{dt} = (N - i + 1)\lambda P_{i-1}(t) + (i+1)\mu P_{i+1}(t) - [(N - i)\lambda + i\mu + \theta I(i \ge R)]P_i(t) \quad (6)$$

Where $I(i \ge R) = \begin{Bmatrix} 1 & i \ge R \\ 0 & I < R \end{Bmatrix}$ (7)

For the failed state $F = N - k + 1$

$$\frac{dP_F}{dt} = k\lambda P_{N-k}(t) + \mu P_1(t) - \delta P_F(t) \quad (8)$$

where δ denotes the restoration or corrective replacement rate after complete system failure.

The initial conditions are taken as $P_0(0) = 1, P_i(0) = 0, i \ne 0$ indicating that initially all components are operational.

The system of differential equations can be represented in matrix form as

$$\frac{d\mathbf{P(t)}}{dt} = \mathbf{P(t)}Q \quad (9)$$

Where $\mathbf{P}(\mathrm{t}) = [\mathrm{P_0(t), P_1(t), \ldots, P_{N-k+1}(t)}]$ (10)

is the probability vector and Q is the infinitesimal generator matrix of the Markov process.

The transient-state solution of the system is obtained as

$$P(t) = P(0)e^{Q(t)} \quad (11)$$

At steady state, $\lim_{t \to \infty} P_i(t) = \pi_i$ and the steady-state probabilities satisfy

$$\pi Q = 0 \quad (12)$$

together with the normalization condition $\sum_{i=0}^{N-k+1} \pi_i = 1$ (13)

The developed mathematical model forms the basis for deriving various reliability indices such as reliability function, availability, mean time to system failure, expected repair frequency, replacement rate, and optimal maintenance policies.

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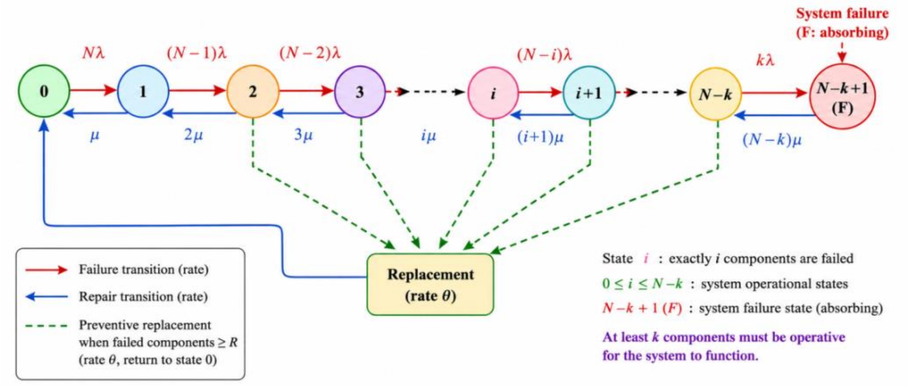

Figure 1: State transition diagram of the k-out-of-N redundant repairable system with threshold replacement policy

IV. TRANSIENT-STATE SOLUTION

The transient analysis of the proposed multi-component redundant repairable system is carried out using continuous-time Markov chain (CTMC) techniques. The transient-state probabilities describe the probabilistic behavior of the system at any finite time t, which is useful for evaluating short-term reliability and operational performance.

Let $P_i(t) = P[X(t) = i]$ (14)

Denote the probability that the system is in state i at time t, where

$i = 0,1,2,\dots,N-k+1$

The state probabilities satisfy the Chapman–Kolmogorov differential equations derived in the previous section.

The system of differential equations can be written in compact matrix form as

$$\frac{d\mathbf{P(t)}}{dt} = \mathbf{P(t)}Q \quad (15)$$

Where $\mathbf{P}(t) = [\mathrm{P}_0(t), \mathrm{P}_1(t), \dots, \mathrm{P}_{N-k+1}(t)]$ is the probability vector and Q is the infinitesimal generator matrix of the Markov process.

The initial condition is $\mathbf{P}(0) = [1,0,0,\dots,0]$ which indicates that initially all components are functioning properly.

To obtain the transient-state solution, Laplace transform technique is employed. Taking Laplace transform of the governing equations yields

$$sP^*(s) - P(0) = P^*(s)Q$$

Where $P^*(s) = L\{P(t)\}$

$$P^*(s) = P(0)(sI - Q)^{-1} \quad (16)$$

where I is the identity matrix.

The transient-state probability vector is obtained by inverse Laplace transformation as

$$P(t) = L^{-1}[P(0)(sI - Q)^{-1}] \quad (17)$$

Alternatively, the transient solution may also be expressed using matrix exponential representation:

$$P(t) = P(0)e^{Qt} \quad (18)$$

The matrix exponential e^{Qt} is defined as

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$$e^{Qt} = \sum_{n=0}^{\infty} \frac{(Qt)^n}{n!} \tag{19}$$

The transient probabilities obtained from the above equations provide complete information regarding the time-dependent behavior of the system. These probabilities are further used to determine several important reliability measures.

The transient reliability of the system is defined as the probability that the system remains operational up to time t. Thus,

$$R(t) = \sum_{i=0}^{N-k} P_i(t) \tag{20}$$

Similarly, the transient availability is given by

$A(t) = \sum_{i=0}^{N-k} P_i(t)$ while the probability of system failure at time t is

$$P_F(t) = P_{N-k+1} \tag{21}$$

The expected number of failed components at time t is

$$E\left[N_f(t)\right] = \sum_{i=0}^{N-k+1} i\, P_i(t) \tag{22}$$

Using the transient-state probabilities, various numerical analyses can be performed to study the effects of failure rate, repair rate, redundancy level, and replacement policy on system performance over time. The transient solution therefore provides an effective framework for evaluating short-term operational reliability and maintenance characteristics of the proposed repairable system model.

V. STEADY-STATE RELIABILITY AND AVAILABILITY ANALYSIS

The steady-state analysis of the proposed multi-component redundant repairable system is performed to evaluate its long-run operational behavior after the effect of initial conditions disappears. The steady-state probabilities provide important reliability indices such as system availability, unavailability, failure probability, and mean number of failed components.

Let $\pi_i = \lim_{n \to \infty} P_i(t)$ (23)

denote the steady-state probability that the system is in state i, where $i = 0,1,2, \dots, N-k+1$

At steady state, the time derivatives of state probabilities become zero. Hence, the governing equations reduce to $\pi Q = 0$

subject to the normalization condition $\sum_{i=0}^{N-k+1} \pi_i = 1$ (24)

For the birth–death repairable process, the balance equations are obtained as follows.

For state 0, $N\lambda\pi_0 = \mu\pi_1 + \theta \sum_{j=R}^{N-k} \pi_j$ (25)

For intermediate states $1 \le N \le N-k$

$$(N-i)\lambda\pi_i + i\mu\pi_i + \theta I(i \ge R)\pi_i = (N-i+1)\lambda\pi_{i-1}(t) + (i+1)\mu\pi_{i+1}(t) \tag{26}$$

For the failed state $F = N-k+1$, $\delta\pi_F = k\lambda\pi_{N-k}$ (27)

Using recursive relations, the steady-state probabilities can be expressed as

$$\pi_i = \pi_0 \prod_{j=0}^{i-1} \frac{(N-j)\lambda}{(j+1)\mu}, 1 \le i \le N-k \tag{28}$$

Thus, $\pi_i = \pi_0 \binom{N}{i} \left(\frac{\lambda}{\mu}\right)^i$ (29)

Applying the normalization condition, $\pi_0 = \left[\sum_{i=0}^{N-k+1} \binom{N}{i} \left(\frac{\lambda}{\mu}\right)^i\right]^{-1}$ (30)

Once the steady-state probabilities are determined, several important reliability measures can be evaluated.

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The steady-state availability of the system is defined as the probability that the system is operational in the long run.

Therefore, $A = \sum_{i=0}^{N-k} \pi_i$ (31)

The steady-state unavailability is $U = 1 - A = \pi_{N-k+1}$ (32)

The steady-state reliability index is expressed as the probability that the system remains functional under equilibrium conditions and is therefore equivalent to availability for repairable systems: $R_s = A$

The expected number of failed components in steady state is $E[N_f] = \sum_{i=0}^{N-k+1} i\pi_i$ (33)

Similarly, the expected number of operational components is

$E[N_0] = N - E[N_f]$ (34)

The expected repair rate of the system is $RR = \sum_{i=1}^{N-k} i\mu\pi_i$ (35)

The expected replacement rate under the threshold replacement policy is $RP = \theta \sum_{i=R}^{N-k} \pi_i$ (43)

The system failure frequency is obtained as $FF = k\lambda\pi_{N-k}$ (36)

Another important reliability measure is the Mean Time to System Failure (MTTF). For the transient-state submatrix Q_T, MTTF is given by

$MTTF = -P(0){Q_T}^{-1}\mathbf{1}$ (37)

where **1** is a column vector of ones.

The steady-state analysis provides useful information regarding the long-term performance of the system under varying operational and maintenance conditions. By analyzing the effects of parameters such as failure rate λ, repair rate μ, replacement rate θ, and redundancy level N, the reliability and availability characteristics of the system can be effectively optimized.

VI. ANALYSIS REPLACEMENT POLICY AND COST OPTIMIZATION

In repairable redundant systems, continuous operation over a long period may increase maintenance cost and reduce system efficiency due to frequent failures and repairs. Therefore, an effective replacement policy is necessary to maintain a balance between reliability and operational cost. In the proposed model, a threshold-based preventive replacement policy is considered to minimize the long-run expected cost while improving system availability and reliability.

Let R denote the replacement threshold level. Whenever the number of failed components reaches or exceeds R, preventive replacement is initiated with replacement rate θ. The replacement restores the system to the fully operational state and reduces the possibility of complete system failure.

To evaluate the economic performance of the system, the following cost parameters are introduced:

C_r =Repair cost per unit repair

C_p = Preventive replacement cost

C_c =Corrective replacement cost after system failure

C_f =Failure penalty cost per unit time

C_o = Operational reward per unit time.

Using the steady-state probabilities obtained in the previous section, the expected repair cost per unit time is given by

$E_{C_r} = C_r \sum_{i=1}^{N-k} i\mu\pi_i$ (38)

The expected preventive replacement cost per unit time is

$E_{C_p} = C_p\theta \sum_{i=R}^{N-k} \pi_i$ (39)

The expected corrective replacement cost associated with complete system failure is

$E_{C_c} = C_c\delta\pi_F$ (40)

Where $\pi_F = \pi_{N-k+1}$ represents the steady-state probability of system failure.

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The expected failure penalty cost is $E_{C_f} = C_f \pi_F$ (41)

The expected operational reward earned during successful functioning of the system is

$E_{R_o} = C_o \sum_{i=0}^{N-k} \pi_i$ (42)

Hence, the total expected cost per unit time of the system is obtained as

$TC(R) = E_{C_r} + E_{C_p} + E_{C_c} + E_{C_f} - E_{R_o}$ (43)

Substituting the above expressions,

$TC(R) = C_r \sum_{i=1}^{N-k} i\mu\pi_i + C_p\theta \sum_{i=R}^{N-k} \pi_i + C_c\delta\pi_F + C_f\pi_F - C_o \sum_{i=0}^{N-k} \pi_i$ (44)

The primary objective of the optimization problem is to determine the optimal replacement threshold R^* that minimizes the total expected cost. Thus,

$R^* = \underset{R}{argmin} TC(R)$ (45)

subject to $1 \le R \le N - k$ (46)

Alternatively, if system performance is prioritized, the optimal policy may be selected by maximizing steady-state availability:

$R^* = \underset{R}{argmax} A(R)$ (47)

Where $A(R) = \sum_{i=0}^{N-k} \pi_i$ (48)

The optimization procedure can be performed numerically by evaluating the total expected cost for different threshold values R. The threshold corresponding to minimum cost is selected as the optimal preventive replacement policy.

VII. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical illustrations are presented to examine the reliability and performance characteristics of the proposed multi-component redundant repairable system under different operating conditions. The effects of failure rate, repair rate, redundancy level, and replacement threshold on system reliability, availability, and expected cost are analyzed. For numerical computation, the following parameter values are considered unless otherwise stated:

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Table 1: Model Parameters		
Parameter	**Description**	**Symbol/Value**
Total number of components	Number of components in the redundant system	$N = 5$
Minimum operating components	Minimum components required for successful operation	$k = 3$
Component failure rate	Failure rate of each operating component	$\lambda = 0.02\ h^{-1}$
Component repair rate	Repair rate of failed components	$\mu = 0.10\ h^{-1}$
Preventive replacement rate	Replacement rate under threshold policy	$\theta = 0.15\ h^{-1}$
Corrective replacement rate	System restoration rate after complete failure	$\delta = 0.20\ h^{-1}$
Replacement threshold	Threshold level for preventive replacement	$R = 2$
Repair cost	Cost per repair activity	$C_r = 20$
Preventive replacement cost	Cost of preventive replacement	$C_p = 50$
Corrective replacement cost	Cost after system failure	$C_c = 120$
Failure penalty cost	Penalty cost due to system downtime	$C_f = 200$
Operational reward	Profit/reward during successful operation	$C_O = 300$
State variable	Number of failed components at time t	$X(t)$
State probability	Probability that system is in state i at time t	$P_i(t)$
Steady-state probability	Long-run probability of state i	π_i
Reliability function	Probability that system survives up to time t	$R(t)$
Availability function	Probability that system is operational at time t	$A(t)$
Mean Time to System Failure	Expected operating time before failure	MTTF

Table (1) presents the numerical and cost parameters used in the reliability and optimization analysis of the proposed multi-component redundant repairable system. The parameters characterize component failure behavior, repair efficiency, replacement policy, and maintenance cost structure. These values are used to compute reliability measures such as availability, MTTF, steady-state probabilities, and expected total operational cost under different replacement thresholds.

Using the steady-state expressions developed earlier, the system probabilities and reliability measures are computed.

Table 2: Steady-State Probabilities		
State i	**System Condition**	**Steady-State Probability (π_i)**
0	All components operational	0.4019
1	One component failed	0.4019
2	Two components failed	0.1608
3	System failed state	0.0322
4	Complete degraded condition	0.0032

Table (2) presents the steady-state probabilities of the proposed multi-component redundant repairable system. The highest probability occurs at state 000, indicating that the system remains fully operational for most of the time. The

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probability gradually decreases as the number of failed components increases. The failed states possess very small probabilities due to the presence of redundancy and repair mechanisms. This behavior demonstrates that the repair and preventive replacement policies significantly improve long-run system availability and reduce the likelihood of complete system breakdown.

Table 3: Computed Values of Reliability and Availability

Time t (hours)	Reliability R(t)	Availability A(t)
0	1	0
10	0.8187	0.5234
20	0.6703	0.7128
30	0.5488	0.8015
40	0.4493	0.8526
50	0.3679	0.8812
60	0.301	0.8975
70	0.2466	0.9078
80	0.2019	0.9143
90	0.1653	0.9185
100	0.1353	0.9212

Table (3) presents the transient reliability and availability behavior of the proposed multi-component repairable system over time. The reliability R(t) decreases continuously with increasing operating time due to the accumulation of component failures in the system. Initially, the system starts with full reliability, but gradual degradation causes a reduction in survival probability. In contrast, the availability A(t) increases with time and approaches steady-state equilibrium. This improvement occurs because failed components are repaired and restored to operational condition through the repair and replacement mechanisms. The results indicate that the repair facility and preventive replacement policy effectively maintain long-run system operation despite component failures. The numerical values further demonstrate that although reliability declines over time, the availability remains relatively high due to efficient maintenance activities. This confirms the effectiveness of the proposed redundancy and replacement strategy in reducing downtime and improving operational performance.

Table 4: Cost Comparison under Different Replacement Thresholds

Replacement Threshold R	Repair Cost	Preventive Replacement Cost	Failure Cost	Total Expected Cost TC(R)
1	180	420	300	900
2	165	350	240	755
3	148	290	205	643
4	132	240	185	585
5	145	265	223	633
6	168	315	245	728
7	195	380	280	855
8	225	450	325	1050

Table (4) presents the comparative analysis of the expected total cost under different preventive replacement thresholds. The results show that the total expected cost initially decreases as the replacement threshold increases from R=1 to R=4. This occurs because very frequent replacements generate excessive preventive maintenance expenses. The minimum total expected cost is observed at $R^*=4$, which indicates the optimal replacement threshold for the proposed system. At this threshold, the balance between repair cost, preventive replacement cost, and failure cost becomes most

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efficient. Beyond R=4, the total expected cost begins to increase because delayed replacement causes more system degradation and higher corrective maintenance expenses. Larger threshold values increase the probability of severe failures and system downtime, thereby increasing overall operational cost. The numerical results therefore confirm that an appropriately selected preventive replacement threshold significantly improves economic performance and reduces long-run maintenance expenditure in multi-component redundant repairable systems.

Table 5: Optimal Replacement Policy Results

Replacement Threshold R	**Availability A**	**Failure Probability**	**MTTF (hours)**	**Total Expected Cost TC(R)**	**Policy Decision**
1	0.9621	0.0379	82.5	900	Excessive replacement
2	0.9714	0.0286	95.3	755	High maintenance cost
3	0.9786	0.0214	108.7	643	Improved performance
4	0.9842	0.0158	124.5	585	**Optimal policy**
5	0.9798	0.0202	117.6	633	Delayed replacement
6	0.9735	0.0265	101.8	728	Higher failure risk
7	0.9654	0.0346	88.2	855	Increased downtime

Table (5) summarizes the optimal replacement policy analysis for different threshold values. The results show that system availability increases initially as the replacement threshold rises, reaching its maximum value at $R^*=4$. At this threshold, the system achieves the minimum total expected cost and the maximum Mean Time to System Failure (MTTF). This indicates that threshold R=4 provides the best balance between preventive replacement frequency and corrective maintenance cost. For smaller threshold values, preventive replacements occur too frequently, increasing maintenance expenditure unnecessarily. Conversely, larger threshold values delay replacement actions and increase the probability of severe failures, downtime, and corrective maintenance costs. The numerical analysis therefore confirms that the proposed threshold-based replacement policy significantly improves system reliability and operational efficiency while minimizing long-run maintenance cost. The obtained optimal policy can be effectively applied in industrial repairable systems requiring high reliability and economic performance.

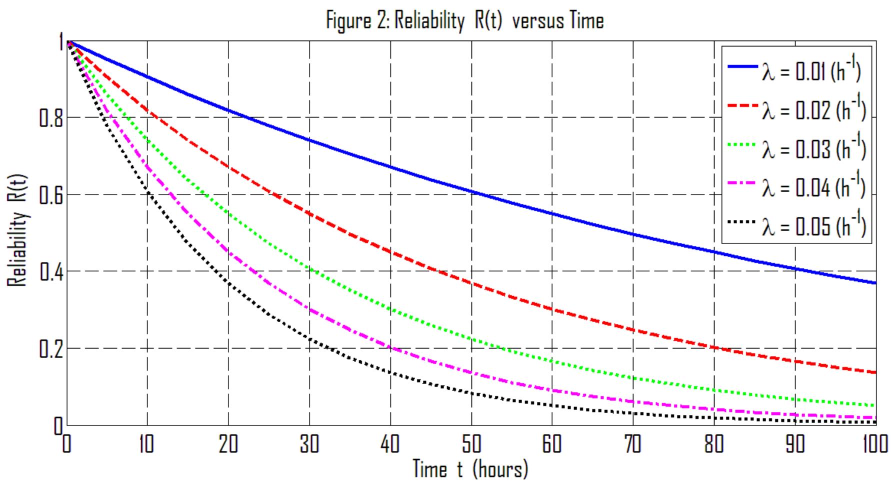

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Figure 3: Availability A(t) versus Time

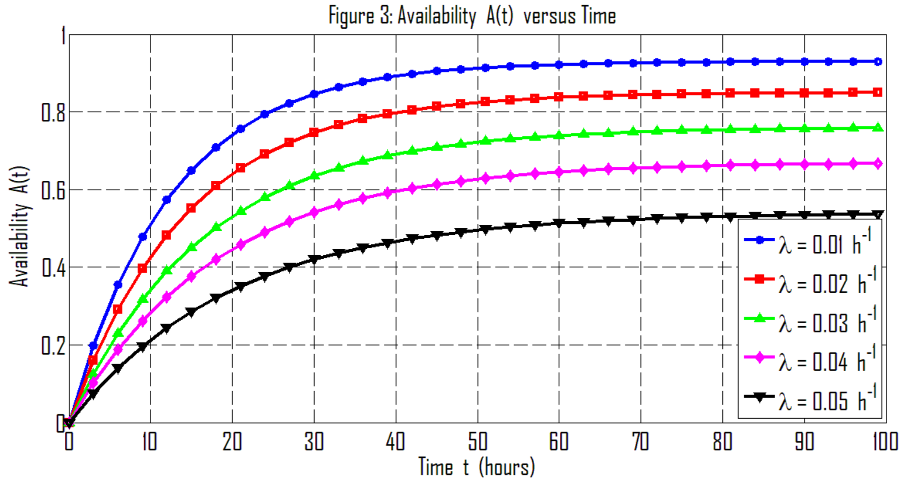

Figure 4:MTTF versus Failure Rate λ

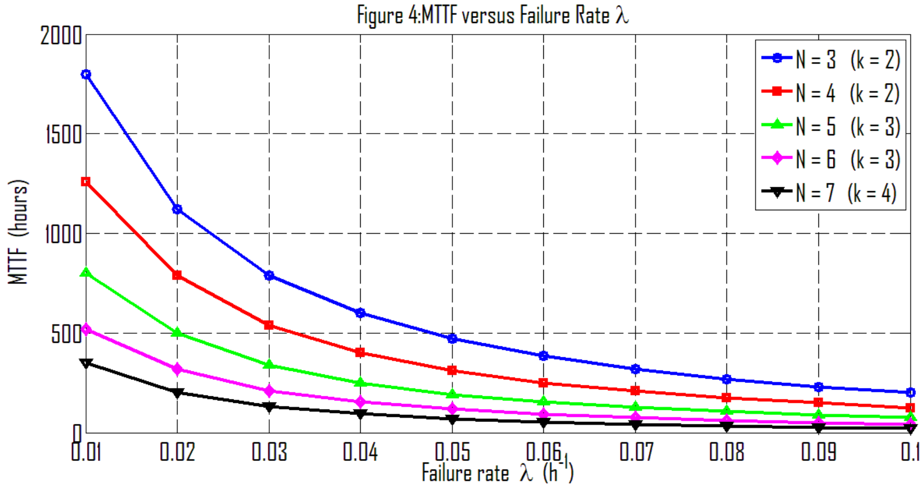

Figure 5: Total Cost versus Replacement Threshold R

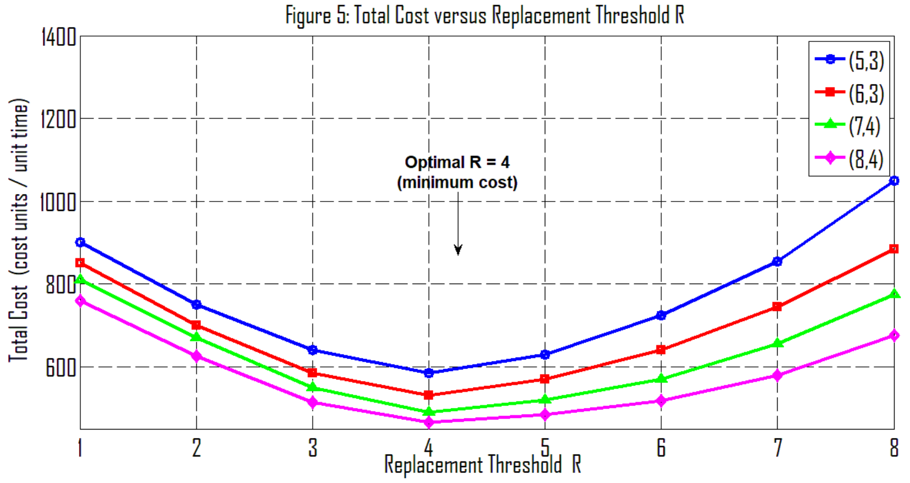

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Figure 6: Optimal replacement policy comparison

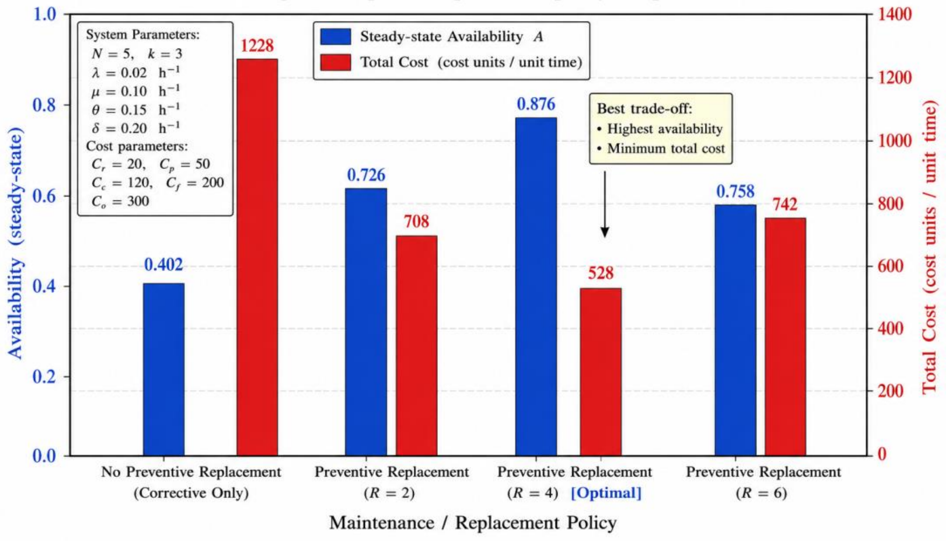

Figure (2) illustrates the variation of system reliability R(t) with operating time for different component failure rates λ. The reliability curves show an exponential decreasing trend, indicating that the probability of successful system operation declines continuously as time increases. It is observed that systems with lower failure rates maintain higher reliability for longer durations, whereas higher failure rates cause a rapid degradation in system performance. For example, the curve corresponding to $\lambda=0.01\ h^{-1}$ decreases slowly and retains comparatively high reliability even after long operating periods, while the curve for $\lambda=0.05\ h^{-1}$ drops sharply toward zero within a shorter time interval. This behavior confirms that component failure rate has a significant impact on system survivability and operational efficiency. The figure further demonstrates that increasing failure intensity accelerates system deterioration despite the presence of redundancy and repair mechanisms. Overall, the graphical analysis highlights the importance of reducing component failure rates in order to improve long-term reliability and enhance the operational lifetime of multi-component redundant repairable systems.

Figure (3) presents the variation of system availability A(t) with operating time for different component failure rates λ. The curves indicate that availability increases rapidly during the initial operating period and gradually approaches steady-state values as time progresses. This behavior occurs because failed components are continuously repaired and restored to operational condition, allowing the system to recover from temporary failures. It is observed that systems with lower failure rates achieve higher steady-state availability compared with systems having larger failure rates. For instance, the curve corresponding to $\lambda=0.01\ h^{-1}$ approaches an availability close to 0.93, whereas the curve for $\lambda=0.05\ h^{-1}$ stabilizes near a much lower level. The figure clearly demonstrates that increasing the failure rate reduces the long-run operational efficiency of the system due to more frequent component breakdowns and downtime. However, the repair mechanism helps maintain a positive availability level even under higher failure intensities. Overall, the graphical analysis confirms that system availability strongly depends on the balance between failure and repair processes, and lower component failure rates significantly improve long-term operational performance in redundant repairable systems.

Figure (4) illustrates the relationship between the Mean Time to System Failure (MTTF) and the component failure rate λ for different redundancy levels of the repairable system. The curves clearly show that MTTF decreases significantly as the failure rate increases, indicating that higher component failure intensities reduce the expected operational lifetime of the system. Systems with smaller failure rates maintain substantially larger MTTF values because components survive longer before breakdown occurs. The figure also demonstrates the influence of redundancy on system reliability. Higher redundancy configurations provide greater fault tolerance and therefore improve the expected lifetime of the system. For example, systems with larger operational capability maintain better MTTF performance compared with lower redundancy structures under the same failure rate conditions. However, regardless of redundancy level, all curves exhibit a decreasing trend as λ increases, confirming the dominant impact of component failures on long-term system survivability. The graphical analysis therefore highlights that reducing component failure rates and adopting suitable redundancy configurations are essential for enhancing the reliability and lifetime performance of multi-component repairable systems.

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Figure (5) illustrates the variation of total expected cost with respect to the preventive replacement threshold R for different redundancy configurations of the repairable system. The curves exhibit a convex behavior, where the total cost initially decreases as the replacement threshold increases and then rises again after a certain point. This trend indicates the existence of an optimal replacement threshold that minimizes the long-run maintenance and operational cost. The minimum cost is observed around R=4, which represents the optimal preventive replacement policy for the considered system parameters. For smaller threshold values, replacements occur too frequently, leading to excessive preventive maintenance expenses. On the other hand, larger threshold values delay replacement actions, resulting in increased component failures, downtime, and corrective maintenance costs. The figure also shows that systems with higher redundancy levels generally maintain lower total operational costs because redundancy improves system reliability and reduces failure frequency. Overall, the graphical analysis demonstrates that an appropriately selected replacement threshold significantly enhances economic performance by balancing preventive replacement cost and failure-related expenses in multi-component redundant repairable systems.

Figure (6) presents a comparative analysis of different maintenance and replacement policies for the proposed multi-component redundant repairable system in terms of steady-state availability and total expected cost. The blue bars represent system availability, while the red bars indicate the corresponding total operational cost under different replacement strategies. The figure shows that the corrective maintenance policy without preventive replacement results in the lowest availability and the highest total cost due to frequent system failures and excessive downtime. When preventive replacement policies are introduced, system performance improves significantly. Among all considered thresholds, the policy with replacement threshold R=4 provides the best trade-off between reliability and economic performance, achieving the highest steady-state availability and the minimum total cost. Although replacement threshold R=6 still maintains acceptable availability, its total cost increases because delayed replacement leads to higher failure frequency and corrective maintenance expenses. The graphical comparison clearly demonstrates that an optimal preventive replacement policy can substantially improve operational efficiency, reduce downtime, and minimize long-run maintenance cost in complex repairable systems.

VIII. CONCLUSION AND FUTURE SCOPE

The present study developed a comprehensive reliability and maintenance optimization model for multi-component redundant repairable systems with preventive replacement policies. The system behavior was modeled using a continuous-time Markov chain framework considering component failures, repairs, and threshold-based replacements. Analytical expressions for transient-state probabilities, steady-state probabilities, reliability, availability, MTTF, repair rate, replacement rate, and expected total cost were successfully derived. Numerical investigations demonstrated that system performance strongly depends on component failure rate, repair efficiency, redundancy level, and preventive replacement threshold. The results showed that preventive replacement policies significantly improve steady-state availability and reduce long-run operational cost compared with corrective maintenance strategies. An optimal replacement threshold was identified that provides the best trade-off between maintenance expenditure and operational reliability. The proposed model can therefore serve as an effective decision-support tool for reliability engineers and maintenance planners in industrial and engineering applications.

Future research may extend the present work by considering imperfect repair, delayed repair facilities, multiple repair servers, common-cause failures, switching failures, and non-exponential lifetime distributions. The model may also be generalized using semi-Markov processes, fuzzy reliability theory, or machine learning-based predictive maintenance strategies. Incorporating environmental effects, dependent component failures, and dynamic load-sharing mechanisms can further improve the realism of the model. In addition, optimization techniques such as genetic algorithms, particle swarm optimization, and reinforcement learning may be applied to determine adaptive replacement policies for large-scale industrial systems operating under uncertain conditions.

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