IARJSET ISSN (O) 2393-8021, ISSN (P) 2394-1588

International Advanced Research Journal in Science, Engineering and Technology

Impact Factor 8.311 ⵗ Peer-reviewed & Refereed journal ⵗ Vol. 13, Issue 3, March 2026

DOI: 10.17148/IARJSET.2026.13326

# SPECIAL RECTANGLES WITH 3-DIGIT SPY AND AUTOMORPHIC NUMBERS

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**Abstract:** This study identifies the rectangles with dimensions $x$ and $y$ such that the expression $x^2+y^2+3A-S^2+k^2+Sk$, $k \geq 0$ is represented by $3-$ digit Spy number and Automorphic number. A and S denotes the area and semi-perimeter of the rectangle. Total number of rectangles satisfying the above relations are obtained.
Furthermore, both primitive and non-primitive rectangles are accounted for in the total tally of solutions derived from the proposed relationship.

**Keywords:** Automorphic number, primitive, non-primitive and Spy number.

## I. INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number Theory is one of the largest and oldest branches of Mathematics. The main goal of Number Theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In Number Theory, rectangles have been a matter of interest to various mathematicians.

For more ideas and interesting facts one can refer [1,2]. Recently in [3] special pythagorean triangles in connections with Narcissistic numbers are obtained. [4-10] was referred for connections between Special rectangles and polygonal numbers, Jarasandha numbers. For an extensive variety of fascinating problems, one may refer [11,12]. Apart from the polygonal numbers, we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [13-15].
This study identifies for infinitely many rectangles such that $x^2+y^2+3A-S^2+k^2+Sk$, $k \geq 0$ is represented by $3-$ digit Spy number and Automorphic number respectively, in which $x$ and $y$ represents the length and breadth of the rectangle.
Also the total number of rectangles satisfying the relation under consideration as well as primitive and non-primitive rectangles are also explored.

## II. NOTATIONS

A - Area of the rectangle
S - Semi-perimeter of the rectangle

## III. BASIC DEFINITIONS

**Definition 1: Primitive Rectangle**
A rectangle is said to be primitive if $u, v$ are of opposite parity and $\gcd(u,v)=1$, where $x=u+v$, $y=u-v$ & $u>v>0$.

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**Definition 2: Non-primitive Rectangle**

A rectangle is said to be non- primitive if $u, v$ are of opposite parity and $\gcd(u,v) \neq 1$.

**Definition 3: Automorphic Number**

An automorphic number is a number whose square ends with the number itself, meaning the last digits of the square match the original number, like $a$ $\left(a^2 = na\right)$, where $a$ is an automorphic number, $n$ is an integer.

**Example:**

$6$ $\left(6^2 = 36\right)$ and $76$ $\left(76^2 = 5776\right)$.

**Definition 4: Spy Number**

A spy number is a number where the sum of its individual digits equals the product of its digits, $x + y + z = s$, $x \times y \times z = s$, where $s$ is a spy number and $x, y, z$ are an integer.

**Example:**

$123$ is a spy number because $(1+2+3=6)$ and $(1\times 2\times 3=6)$.

$1124$ is also a spy number $(1+1+2+4=8)$ and $(1\times 1\times 2\times 4=8)$.

## IV. METHOD OF ANALYSIS

**Case (a) Consider 3-digit Spy numbers**

Let $x$ and $y$ be two non-zero distinct positive integers representing the length and breadth of a rectangle $R$. Let $k \geq 0$ be any given integer.

The problem under consideration is to solve the equation

$$x^2 + y^2 + 3A - S^2 + k^2 + Sk = 3-\text{digit Spy number} \qquad (1)$$

To solve $(1)$, let us introduce the linear transformation $x = u + v$ and $y = u - v$ $(u > v > 0)$ $\qquad (2)$

Therefore equation $(1)$, reduces to

$$(u+k)^2 - v^2 = 3-\text{digit Spy number} \qquad (3)$$

**Case (1):**

Consider the $3-$ digit Spy number $123$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 123 \qquad (4)$$

By the method of factorization, the following results were obtained

Table: 1.1

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 62 | 61 | 123 |
|  | 22 | 19 | 123 |
| 1 | 21 | 19 | 123 |
| 2 | 20 | 19 | 123 |

 

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From the above table, the following results were observed.

Table: 1.2

| $k$ | No.of Rectangles related to $123$ | Observations |
|---|---|---|
| 0 | 2 | All the $2-$Rectangles are primitive . |
| 1,2 | 1 | For $k=1,2,$ the Rectangles are primitive. |

**Case (2):**

Consider the $3-$ digit Spy number $132$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 132 \tag{5}$$

After applying the method of factorization, one may have

Table: 1.3

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 34 | 32 | 132 |
| | 14 | 8 | 132 |
| 1 | 33 | 32 | 132 |
| | 13 | 8 | 132 |
| 2 | 12 | 8 | 132 |
| 3 | 11 | 8 | 132 |
| 4 | 10 | 8 | 132 |
| 5 | 9 | 8 | 132 |

From the above table, the following observation were presented here.

Table: 1.4

| $k$ | No.of Rectangles related to $132$ | Observations |
|---|---|---|
| 0,1 | 2 | For $k=0,$ All the $2-$Rectangles are non-primitive.<br>For $k=1,$ All the $2-$Rectangles are primitive. |
| 2,3,4,5 | 1 | For $k=2,4,$ the Rectangles are non-primitive.<br>For $k=3,5,$ the Rectangle is primitive. |

**Case (3):**

Consider the $3-$ digit Spy number $213$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 213 \tag{6}$$

By applying the method of factorization, yields

  

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Table: 1.5

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 107 | 106 | 213 |
| | 37 | 34 | 213 |
| 1 | 36 | 34 | 213 |
| 2 | 35 | 34 | 213 |

From the above mentioned values, the following results were observed.

Table: 1.6

| $k$ | No. of Rectangles related to $213$ | Observations |
|---|---|---|
| 0 | 2 | All the $2-$Rectangles are primitive. |
| 1,2 | 1 | For $k=1$, the Rectangle is non-primitive.<br>For $k=2$, the Rectangle is primitive. |

**Case (4):**

Consider the $3-$ digit Spy number $231$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 231 \qquad (7)$$

By applying the method of factorization, the following table presented the values

Table: 1.7

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 116 | 115 | 231 |
| | 40 | 37 | 231 |
| | 20 | 13 | 231 |
| | 16 | 5 | 231 |
| 1 | 39 | 37 | 231 |
| | 19 | 13 | 231 |
| | 15 | 5 | 231 |
| 2 | 38 | 37 | 231 |
| | 18 | 13 | 231 |
| | 14 | 5 | 231 |
| 3 | 17 | 13 | 231 |
| | 13 | 5 | 231 |
| 4 | 16 | 13 | 231 |
| | 12 | 5 | 231 |
| 5 | 15 | 13 | 231 |
| | 11 | 5 | 231 |
| 6 | 14 | 13 | 231 |
| | 10 | 5 | 231 |
| 7 | 9 | 5 | 231 |
| 8 | 8 | 5 | 231 |
| 9 | 7 | 5 | 231 |
| 10 | 6 | 5 | 231 |

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From the above mentioned values, the following results were observed.

Table: 1.8

| $k$ | No.of Rectangles related to 231 | Observations |
|---|---|---|
| 0 | 4 | All the $4-$Rectangles are primitive . |
| 1,2 | 3 | For $k=1$, the one rectangle is non-primitive and another two rectangles are primitive.<br>For $k=2$, All the $3-$ Rectangles are primitive. |
| 3,4,5,6 | 2 | For $k=3,4$, All the $2-$Rectangles are primitive.<br>For $k=5,6$, the one rectangle are primitive and another rectangle are non-primitive. |
| 7,8,9,10 | 1 | For $k=7,8,9,10$, the rectangles are primitive. |

**Case (5):**

Consider the $3-$ digit Spy number $312$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 312 \qquad (8)$$

By applying the method of factorization, one may have values of $u$ and $v$ are presented following table

Table: 1.9

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 79 | 77 | 312 |
| | 41 | 37 | 312 |
| | 29 | 23 | 312 |
| | 19 | 7 | 312 |
| 1 | 78 | 77 | 312 |
| | 40 | 37 | 312 |
| | 28 | 23 | 312 |
| | 18 | 7 | 312 |
| 2 | 39 | 37 | 312 |
| | 27 | 23 | 312 |
| | 17 | 7 | 312 |
| 3 | 38 | 26 | 312 |
| | 26 | 23 | 312 |
| | 16 | 7 | 312 |
| 4 | 25 | 23 | 312 |
| | 15 | 7 | 312 |
| 5 | 24 | 23 | 312 |
| | 14 | 7 | 312 |
| 6 | 13 | 7 | 312 |
| 7 | 12 | 7 | 312 |
| 8 | 11 | 7 | 312 |
| 9 | 10 | 7 | 312 |
| 10 | 9 | 7 | 312 |
| 11 | 8 | 7 | 312 |

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From the above table, the following results were observed and presented in table 1.10.

Table: 1.10

| $k$ | No.of Rectangles related to $312$ | Observations |
|---|---|---|
| 0,1 | 4 | For $k=0,1,$ All the $4-$Rectangles are primitive. |
| 2,3 | 3 | For $k=2,3,$ All the $3-$Rectangles are primitive. |
| 4,5 | 2 | For $k=4,$ All the $2-$Rectangles are primitive. For $k=5,$ the one rectangle is primitive and another rectangle is non-primitive. |
| 6,7,8,9,10,11 | 1 | For $k=6,7,8,9,10,11,$ the rectangles are primitive. |

**Case (6):**
Consider the $3-$ digit Spy number $321$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 321 \qquad (9)$$

Applying the method of factorization, one may have

Table: 1.11

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 161 | 160 | 321 |
| | 55 | 52 | 321 |
| 1 | 54 | 52 | 321 |
| 2 | 53 | 52 | 321 |

From the above table, the following results were observed.

Table: 1.12

| k | No.of Rectangles related to $321$ | Observations |
|---|---|---|
| 0 | 2 | All the $2-$Rectangles are primitive. |
| 1,2 | 1 | For $k=1,$ the rectangle is non-primitive. For $k=2,$ the rectangle is primitive. |

**Case (b) Consider 3-digit Automorphic numbers**

Let $x$ and $y$ be two non-zero distinct positive integers representing the length and breadth of a rectangle $R$. Let $k \geq 0$ be any given integer.

The problem under consideration is to solve the equation

$$x^2 + y^2 + 3A - S^2 + k^2 + Sk = 3- \text{digit Automorphic number} \qquad (1)$$

To solve $(1)$, let us introduce the linear transformation $x = u + v$ and $y = u - v$ $(u > v > 0)$ $\qquad (2)$

Therefore equation $(1)$, reduces to

DOI: 10.17148/IARJSET.2026.13326

$$(u+k)^2 - v^2 = 3-\text{digit Automorphic Number} \quad (3)$$

**Case (1):**

Consider the $3-$ digit Automorphic number $376$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 376 \quad (4)$$

Applying the method of factorization, yields

Table: 1.13

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 95 | 93 | 376 |
|  | 49 | 45 | 376 |
| 1 | 94 | 93 | 376 |
|  | 48 | 45 | 376 |
| 2 | 47 | 45 | 376 |
| 3 | 46 | 45 | 376 |

From the above mentioned values, the following results were observed.

Table: 1.14

| $k$ | No.of Rectangles related to $376$ | Observations |
|---|---|---|
| 0,1 | 2 | For $k=0$, All the $2-$Rectangles are primitive.<br>For $k=1$, One Rectangle is primitive and another one is non-primitive. |
| 2,3 | 1 | For $k=2,3$, the Rectangles are primitive. |

**Case (2):**

Consider the $3-$ digit Automorphic number $625$.

Equation $(3)$ becomes,

$$(u+k)^2 - v^2 = 625 \quad (5)$$

By applying the method of factorization, the following results were presented here

Table: 1.15

| $k$ | $u$ | $v$ | $(u+k)^2 - v^2$ |
|---|---|---|---|
| 0 | 313 | 312 | 625 |
|  | 65 | 60 | 625 |
| 1 | 64 | 60 | 625 |
| 2 | 63 | 60 | 625 |
| 3 | 62 | 60 | 625 |
| 4 | 61 | 60 | 625 |

From the above table, the following results were observed.

DOI: 10.17148/IARJSET.2026.13326

Table: 1.16

| $k$ | No.of Rectangles related to $625$ | Observations |
|---|---|---|
| 0 | 2 | One Rectangle is primitive and another one is non-primitive. |
| 1,2,3,4 | 1 | For $k=1,2,3,$ the Rectangles are non-primitive.<br>For $k=4,$ the Rectangle is primitive. |

## V. CONCLUSION

To conclude, this study explores the relationship between rectangles with $3-$digit Spy number and Automorphic number. It also suggests the possibility of identifying similar connections with other special numbers and number patterns in Mathematics.

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