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FIELD THEORY IN PINEAPPLE SPIRALS AND DIGITAL SIGNALS

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Abstract: In this study, we present the mathematical design in Pineapple spirals and digital signals using field theory. The golden Angle and Fibonacci sequence explain the packing in natural spirals. Finite fields and recurrence relations are applied to describe these arrangements mathematically.

Keywords: Finite fields (Galois fields), Fibonacci sequence, Golden angle, Phyllotaxis, Pineapple spirals, Recurrence relations, Digital signals, Signal processing, Pseudo-random sequences, Cryptography, Packing efficiency, Phase mapping **AMS Mathematics Classification:** 92C80, 94A55, 11A07, 11T30, 11B39.

INTRODUCTION

The study of spirals in plants has long been a subject of mathematical and biological interest. Early observations on pineapple spirals were made by Linford [7], followed by Onderdonk [8], who connected these arrangements to the Fibonacci sequence. Hoggatt [3] and Burton [2] further developed the theory of Fibonacci and number sequences, showing their relevance in natural growth and number theory. Jean [4] and Adler [1] provided systemic studies of phyllotaxis and models of contact pressure, establishing the biological foundation for spiral morphogenesis.

In mathematics, finite fields as presented by Lidl and Niederreiter [5] offer a framework for recurrence relations that explain spiral structures with precision. These concepts extend into engineering, where Lin and Costello [6] applied them to error control coding, and Proakis and Manolakis [9] demonstrated their role in digital signal processing. Stallings [10] emphasized their importance in cryptography and secure communication systems.

By linking natural spirals with digital signals, this project builds on a rich body of literature that spans biology, mathematics, and engineering. The comparison of sunflower and pineapple spirals with modern communication techniques illustrates how universal mathematical principles unify natural beauty with technological innovation.

PRELIMINARIES

Definition 1.1. A field is a set of elements together with two operations - addition and multiplication - that satisfy the following properties

1. **Closure:** Adding or multiplying any two elements in the field gives another element in the field.
2. **Associativity:** $(a+b)+c = a+(b+c)$ and $(a\cdot b)\cdot c = a\cdot(b\cdot c)$.
3. **Commutativity:** $a+b = b+a$ and $a\cdot b = b\cdot a$.
4. **Identity elements:** There is an additive identity (0) and a multiplicative identity (1).
5. **Inverses:**
   - For every element a, there exists $-a$ such that $a+(-a) = 0$.
   - For every nonzero element a, there exists $a-1$ such that $a\cdot a-1=1$.
6. **Distributivity:** $a\cdot(b+c) = a\cdot b+a\cdot c$

Definition 1.2. The Fibonacci sequence is a series of numbers where each term is the sum of the two preceding terms. Mathematically,

$$F_{n+2} = F_{n+1}+F_n \text{ with } F_0=0, F_1=1$$

So, The sequence begins: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

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Definition 1.3. A recurrence relation is a mathematical rule that defines each term of a sequence using one or more of its previous terms. It is a way of generating sequences step-by-step rather than writing them explicitly. General form

$$a_{n+1} = f(a_n, a_{n-1}, \dots)$$

Definition 1.4. A pattern is a repeated or structured arrangement of elements that follows a specific rule or principle. Patterns can appear in nature, mathematics and technology, showing order and predictability even when they look complex.

FIELD IN PINEAPPLE AND DIGITALS

2.1 FIELD THEORY IN PINEAPPLE SPIRALS

The arrangement of pineapple scales follows spiral families whose counts are Fibonacci numbers, and this pattern can be modeled using field theory. Each scale can be treated as a point in a discrete field, with its position determined by modular arithmetic and the golden angle of about 137.5°. This angle ensures that new scales are placed in the largest available gaps, producing uniform packing and minimizing overlap.

2.1.1 Golden Ratio

Full Pineapple circle is 360^0. The Golden Ratio comes from dividing a line in two parts,

$$\frac{whole\ length}{long\ part} = \frac{long\ part}{shorter\ part} \qquad \text{----- (1)}$$

Assume, long part =1, short part = x, whole part = 1+x, by (1),

$$\frac{1+x}{1} = \frac{1}{x}$$

This leads to

$$x^2 + x - 1 = 0$$

Solving, we have

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

Since the length are positive

$$x = \frac{-1+\sqrt{5}}{2}$$

The Golden ratio is the ratio of long part to short part

$$\varphi = \frac{1}{x}$$

$$\Rightarrow \quad \phi = \frac{2}{\sqrt{5}-1}$$

$$\Rightarrow \quad \phi = 1.618$$

2.1.2 Golden angle

The golden angle is derived directly from the golden ratio. A full circle consist of 360^0. Divide it according to the golden ratio.

- Larger arc = $\frac{360^0}{\varphi} = 222.5^0$
- Smallar arc = $360^0 - 222.5^0 = 137.5^0$

The smaller arc (137.5^0) is golden angle. Plants use this angle to place new leaves, seeds or scales. So, they spread evenly without overlapping.

Theorem: 2.1

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The golden angle α satisfies

$$\alpha = 2\pi\,(1 - \frac{1}{\varphi}) = \frac{2\pi}{\varphi^2}$$

Proof:

Partition the circle so that the larger arc is $\frac{2\pi}{\varphi}$, The smaller arc is then,

$$\alpha = 2\pi - \frac{2\pi}{\varphi} \text{ ---------(1)}$$

by using, $\varphi^2 = \varphi + 1$

$$1 - \frac{1}{\varphi} = \frac{1}{\varphi^2}$$

Substitute in (1), we get

$$\alpha = \frac{2\pi}{\varphi^2}.$$

Hence, proved.

2.1.3 PHYLLOTAXIS FORMULA (OR) RADIAL GROWTH

Keep the equal area spirit with the discrete law,

$$r_k = a\sqrt{k} \text{ for } k=0,1,2,\ldots$$

where, r_k = distance from center, k = index number, a = scale factor,
and θ_n = angle between two spirals, if a is large then points are apart and a is small then points are tighter.

2.1.4 FIBONACCI SPIRALS

Each pineapple shows three different spiral direction,

- Clockwise spirals
- Anticlockwise spirals
- Vertical/diagonal (sometimes called the 'third family')

These are consecutive terms in Fibonacci sequence

1,1,2,3,5,8,13,21, …

2.1.5 Why are there 3 spirals sets

Because each pineapple scale (eye) is a part of three overlapping spirals,

- One going upward to right
- One going upward to left
- One going almost vertically.

The arrangement gives tight packing and maximum space efficiency.

2.1.6 Spirals appear naturally

- Just like sunflower spirals emerge from the golden angle, pineapple spirals emerge from the field recurrence**.**
- The spirals you see correspond to residue classes in the field (like Fibonacci numbers modulo p).
- As the sequence cycles, different spiral counts dominate, and transitions create small irregularities — similar to sunflower junctions.

2.1.7 Field Index

Let the index be a sequence $\{x_k\}$ in the finite field GF(P) with prime p.

Spirals follow **Fibonacci numbers**: 5, 8, 13, 21…These numbers can be studied in finite fields because they cycle when taken modulo a prime.

Example: Fibonacci sequence modulo 7: 0,1,1,2,3,5,1,6,0, …

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2.1.8 Angle Map between the spirals

Embed field values to phase uniformly,

$$\theta_k = \frac{2\pi}{p}(c_1X_k+c_2),\ c_1 \neq 0, \text{ where}$$

- $\frac{2\pi}{p}$ splits the full circle into p equal slices.
- X_k**:** Picks which slice (phase) we're in, based on the sequence value.
- $c_1 \neq 0$**:** Ensures distinct field values map to distinct phase steps; if $c_1=0$, all angles would collapse to the same value.
- c_2**:** A constant rotation (just turns the whole pattern without changing relative positions).

This makes additions in GF(p) correspond to rotations on the circle, paralleling modular angle.

2.1.9 DISTANCE FORMULA

The formula

$$d_{k,l}^2 = r_k^2 + r_l^2 - 2r_k r_l \cos(\theta_k - \theta_l)$$

is used whenever we want to know **how close two pineapple scales (or "seeds") are to each other** in the spiral arrangement.

Theorem:2.2

For points (r_k, Θ_k) and (r_l, Θ_l), the squared Euclidean distance is

$$d_{k,l}^2 = r_k^2 + r_l^2 - 2r_k r_l \cos(\Theta_k - \Theta_l)$$

Proof

Consider two points in polar coordinates (r_k, Θ_k) and (r_l, Θ_l).Convert this polar coordinates to cartesian $x_k = r_k cos\Theta_k$, $y_k = r_k sin\Theta_k$.Hence, the distance formula is,

$$\begin{aligned} d_{k,l}^2 &= (x_k - x_l)^2 + (y_k - y_l)^2 \\ &= (r_k cos\Theta_k - r_l cos\Theta_l)^2 + (r_k sin\Theta_k - r_l sin\Theta_l)^2 \\ &= r_k^2 (cos^2\Theta_k + sin^2\Theta_k) + r_l^2 (cos^2\Theta_l + sin^2\Theta_l) - 2r_k r_l \cos(\Theta_k - \Theta_l) \\ d_{k,l}^2 &= r_k^2 + r_l^2 - 2r_k r_l \cos(\Theta_k - \Theta_l) \end{aligned}$$

by the cosine difference identity.

Example:2.1.9.1

Let the two points in polar coordinates are given by

$$P_k=(r_k,\theta_k)=(3,45^0),P_l=(r_l,\theta_l)=(4,120^0)$$

find the Euclidean distance $d_{k,l}$ between the two points.

solution

Let us choose two points in polar coordinates

- $P_k = (r_k, \Theta_k) = (3,45^0)$
- $P_l = (r_l, \Theta_l) = (4,120^0)$

By the distance formula,

$$d_{k,l}^2 = 3^2+4^2-2\cdot3\cdot4\cos(45^0-120^0) \text{ --------(1)}$$

Since,

$$\theta_k - \theta_l = 45^0-120^0 = -75^0 \text{ and } \cos(-75^0) = \cos(75^0).$$

(1) Becomes,

$$d_{k,l}^2 = 9+16-24\cos(75^0)$$

$\text{Cos}(75^0) \approx 0.2588$

$$d_{k,l}^2 \approx 25-24(0.2588)$$

$$\approx 25-6.21=18.79$$

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Therefore, $d_{k,l} \approx 4.33$

The two points in polar coordinates are about **4.33 units apart.**

Theorem: 2.3

Let $\Theta_k = \frac{2\pi}{p}(c_1 X_k + c_2)$ with $c_1 \neq 0$. For fixed k, the distance $d_{k,l}$ is minimized (among l near k) when the fixed difference $\triangle X = X_k - X_l$ lies in a small residue class, i.e., $|\triangle X|$ (interpreted as the least absolute residue mod p) is small. In that case, $\triangle \Theta = \frac{2\pi}{p} c_1 \triangle X$ is small, $\cos(\triangle\Theta) = 1$ and

$$d_{k,l}^2 = (r_k - r_l)^2$$

Proof

From theorem 3.3, $d_{k,l}^2 = r_k^2 + r_l^2 - 2r_k r_l \cos(\triangle \theta)$, where $\triangle\Theta = \frac{2\pi}{p} c_1 \triangle X$.

If $|\triangle X|$ is small, the $|\triangle\Theta|$ is small, so $\cos(\triangle \theta) = 1 - \frac{1}{2}\triangle \theta^2$. Hence,

$$\begin{aligned} d_{k,l}^2 &= r_k^2 + r_l^2 - 2r_k r_l \left(1 - \tfrac{1}{2}\triangle \theta^2\right) \\ &= r_k^2 + r_l^2 - 2r_k r_l + r_k r_l \triangle \theta^2 \\ &= (r_k - r_l)^2 + r_k r_l \triangle \theta^2 \end{aligned}$$

For nearby indices, $r_k = r_l$ and the dominant term is $(r_k - r_l)^2$, minimized when $\triangle \theta$ is small.

Example:2.1.9.2

Let the parameters be given by

- Prime modulus: p=7
- Constants: c1=1,c2=0
- Indices: $X_k=3$, $X_l=2$
- Radii: $r_k=5$, $r_l=5.2$

Compute the Euclidean distance $d_{k,l}$ between the two points.

Solution

Let the given parameters be

- Prime modulus: p=7
- Constants: $c_1=1$, $c_2=0$
- Indices: $X_k=3$, $X_l=2$
- Radii: $r_k=5$, $r_l=5.2$ and compute difference in indices,

$$\Delta X = X_k - X_l = 3-2 = 1$$

Since $|\Delta X|=1$ is small, we expect minimal distance and compute angular difference:

$$\Delta\theta = \frac{2\pi}{7} \cdot 1 \approx 0.897 \text{ radians}$$

By distance formula:

$$\begin{aligned} d_{k,l}^2 &= 5^2 + 5.2^2 - 2\cdot5\cdot5.2 \cos(0.897) \\ &= 25 + 27.04 - 52\cdot0.623 \\ &= 52.04 - 32.40 = 19.64 \\ d_{k,l} &\approx 19.64 \approx 4.43 \end{aligned}$$

Compare with simplified case: Since $|\Delta X|$ is small,

$d_{k,l}^2 \approx (r_k - r_l)^2 = (5-5.2)^2 = 0.04$.

2.1.10 Representing pineapple scales

Each scale on the pineapple can be represented as a point in **polar coordinates**

- r_k = distance from the center (radius).
- θ_k = angle around the pineapple, determined by the field recurrence. So, every scale has coordinates (r_k, θ_k).

To find the straight-line distance between these two points, you convert their polar coordinates into Cartesian coordinates and apply the Pythagorean theorem. The result is exactly the law of cosines:

$$d^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos(\triangle \theta)$$

So, this formula tells you **how close two seeds are** depending on their radii and angular separation.

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2.2 FIELD THEORY IN DIGITAL SIGNALS

2.2.1 Indexing and Sequence Generation

In digital signals, we often work with finite field GF(p) where p is a prime.
Signal sequences (like pseudo – random codes, error – correcting codes, or spread spectrum signals) are generated by recurrence relations inside these fields.

Example: $X_{k+2} = X_{k+1} + X_k$ in GF(p)

In signals, this recurrence produces structured, periodic sequence that can be used for synchronization, coding or spreading information.

2.2.2 Mapping to phases(angles)

Each field element can be mapped to a phase angle of a digital signal

$$\theta_k = \frac{2\pi}{p}(c_1 X_k + c_2)$$

In technology, this mapping is used in phase modulation (PSK) where information is carried by the angle of signal.

Addition in the field corresponds to rotation in phase space.

2.2.3 Equal-spacing / energy distribution

- In signals, we want **equal energy distribution** across the sequence.
- Field-based recurrences ensure that each symbol or chip in the signal contributes evenly, avoiding clustering or bias.
- This is why finite fields are used in **spread-spectrum codes** (like Gold codes or m-sequences).

2.2.4 Distance formula → correlation

- The distance formula in pineapple spirals:
  $d_{k,l}^2 = r_k^2 + r_l^2 - 2 r_k r_l \cos(\theta_k - \theta_l)$
- In signals, this corresponds to **correlation between two signal elements**.
- When the phase difference Δθ is small (from small field residues), the correlation is high → elements align (like arcs in pineapple).
- When different residue classes alternate, correlation patterns shift → this is exactly how **multi-user codes** in CDMA or error-correcting codes work.

2.2.5 Where Field Theory Derivations Appear in Digital Signals

(i) Linear Feedback Shift Registers (LFSRs)

- **Derivation:** Sequences are generated by recurrence relations over finite fields:
  $$X_{k+n} = c_{n-1} X_{k+n-1} + \cdots + c_0 X_k \ \ in\ GF(p)$$
- **Result:** Produces *maximum length sequences* (m-sequences) with period p^n-1.
- **Pattern meaning:** These sequences look random but are deterministic, with uniform distribution and predictable correlation properties.
- **Use:** Spread-spectrum codes in CDMA, GPS signals.

(ii) **Gold Codes**

- **Derivation:** Built by combining two m-sequences from finite fields.
- **Result:** Families of sequences with controlled cross-correlation.
- **Pattern meaning:** Multiple users can transmit simultaneously with minimal interference.
- **Use:** Mobile communication (3G, 4G), Wi-Fi.

(iii) **Error-Correcting Codes (Reed–Solomon, BCH)**

- **Derivation:** Codewords are polynomials over GF (p^m).
  - Example: Reed–Solomon codes use evaluation of polynomials at field elements.
- **Result:** Structured redundancy patterns that allow detection and correction of errors.
- **Pattern meaning:** The "arc transitions" in sunflower analogy correspond to syndrome patterns in coding.
- **Use:** DVDs, QR codes, satellite communication.

(iv) **Phase and Frequency Patterns (PSK/QAM)**

- **Derivation:** Field elements mapped to constellation points:
  $$\theta_k = \frac{2\pi}{p}(c_1 X_k + c_2)$$
- **Result:** Uniform phase distribution ensures equal energy and minimal interference.

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- **Pattern meaning:** Just like pineapple spirals, small field differences yield close phase neighbors, forming correlation arcs.
- **Use:** Modulation schemes in digital communication.

CONCLUSION

This project highlights how mathematical concepts such as the Fibonacci sequence, golden angle and finite fields explain natural spirals and extend into modern technology. By connecting pineapple patterns with digital signals, it demonstrates the unity between biology and mathematics. The study shows practical applications in agriculture for understanding crop growth in engineering for improving communication systems and in computer science for enhancing data security. These insights prove that mathematics is not limited to theory but serves as a powerful tool for solving real-world problems.

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