

Similarity Solution of MHD Power-Law Flow Past Accelerated Plate

Trupti R Raypurkar¹, Pankaj Sonawane²

Department of Mathematics, Dhanaji Nana Mahavidyalaya, Faizpur, India^{1,2}

Abstract: The present study investigates the influence of an applied magnetic field on the boundary-layer characteristics of an electrically conducting fluid in a magnetohydrodynamic (MHD) flow configuration. By employing similarity transformations, the governing equations are reduced to a set of dimensionless ordinary differential equations and solved for various values of the magnetic parameter S_0 . The effects of the magnetic field on the dimensionless velocity profile $f(\eta)$ and velocity-gradient profile ($f'(\eta)$) are analyzed in detail. The results reveal that increasing the magnetic parameter significantly suppresses the fluid velocity due to the action of the Lorentz force, which opposes the fluid motion. Consequently, the momentum boundary layer becomes thinner as the magnetic field strength increases. Furthermore, the magnitude of the velocity gradient at the wall increases with increasing magnetic parameter, indicating an enhancement in wall shear stress and skin friction. The study demonstrates that the applied magnetic field provides an effective mechanism for controlling flow behavior and modifying boundary-layer characteristics in electrically conducting fluids.

Keywords: Similarity solution, Group transformation technique, Magnetohydrodynamic flow, non-Newtonian fluid.

INTRODUCTION

The study of fluid flow in the presence of magnetic fields referred to as magnetohydrodynamics (MHD) has garnered significant attention due to its broad applications in industrial processes, geophysics, and astrophysics. In particular, MHD flows involving non-Newtonian fluids, such as power-law fluids, are highly relevant in industries involving complex rheological fluids, such as polymer extrusion, drilling muds, and blood flow modeling. Understanding these flows is essential for optimizing heat transfer systems, electromagnetic pumps, nuclear reactor cooling, and materials processing.

The interaction between a conducting fluid and a magnetic field leads to the emergence of Lorentz forces, which act perpendicular to both the magnetic field and the direction of electric current. These forces can either retard or enhance the flow, depending on the alignment of the magnetic field. The inclusion of MHD effects modifies the momentum equation and introduces additional damping effects due to electromagnetic forces, thus altering the flow and heat transfer characteristics. In practical applications, such as MHD generators, liquid metal cooling systems, and magnetic drug targeting, the influence of MHD on non-Newtonian fluids becomes highly significant. Hence, a detailed analysis of such flows provides valuable insight into the design of engineering systems involving electromagnetic control of complex fluids.

The flow past an accelerated flat plate presents an unsteady boundary layer problem, wherein the plate velocity varies with time. This configuration is particularly important in modeling situations where surfaces experience non-uniform acceleration, such as in wind tunnels, spacecraft re-entry, and certain manufacturing processes.

When the plate is suddenly accelerated in a fluid initially at rest, a time-dependent boundary layer develops, and the fluid responds to the momentum diffusion initiated by the plate's motion. For MHD non-Newtonian flows, this problem becomes significantly more complex due to the combined influence of time-dependence, non-linearity of the constitutive relations, and electromagnetic effects.

To simplify the highly nonlinear partial differential equations governing unsteady MHD flow of power-law fluids, similarity transformations are employed. These transformations reduce the governing equations to a set of ordinary differential equations (ODEs) by collapsing the independent variables into a single similarity variable. The success of this technique hinges on finding suitable transformations that preserve the physics of the original system.

The similarity approach offers several advantages:

- Reduction in computational complexity
- Possibility of obtaining analytical or semi-analytical solutions
- Better understanding of parameter sensitivities and asymptotic behavior

This methodology provides a powerful tool for exploring parametric effects, such as variations in the magnetic field intensity, power-law index, plate acceleration profile, and other non-dimensional groups (e.g., magnetic interaction parameter, Prandtl number).

The primary objective of this study is to derive and analyze a similarity solution for the unsteady, two-dimensional flow of a power-law fluid past an accelerated flat plate in the presence of a transverse magnetic field. The specific goals include:

- Formulating the governing equations incorporating MHD and power-law rheology
- Identifying suitable similarity variables to transform the PDEs into a boundary-value problem involving ODEs
- Analyzing the resulting equations using appropriate numerical or semi-analytical techniques
- Investigating the influence of key parameters such as the power-law index, magnetic field strength, and acceleration function on the velocity and shear stress profiles

This work contributes to the broader field of fluid mechanics by providing new insights into the behavior of electrically conducting non-Newtonian fluids under unsteady motion. The analysis also offers guidance for experimental and numerical investigations in related engineering applications.

Paul's A. [1] studied focuses on the analytical solution of one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in the presence of a transverse magnetic field. The vertical plate and the medium of flow are considered porous, and the fluid is assumed to be optically thin, with a small magnetic Reynolds number to neglect induced hydromagnetic effects.

The study employs the Laplace transform technique to solve the governing boundary layer equations, which are first converted into dimensionless form. The investigation presents numerical values of transient velocity, temperature, skin friction, and Nusselt number, illustrated through graphs for various physical parameters.

Sonawane P.M., Timol M.G. [2], examined the similarity solution for the magnetic Stokes first problem, focusing on an infinite impulsively moving plate with constant velocity in non-Newtonian fluids that extend infinitely. This research investigates the unsteady flow of a magnetohydrodynamic (MHD) boundary layer, where an electrically conducting non-Newtonian fluid is influenced by a transverse external magnetic field.

To establish the similarity equation, the study applies a one-parameter linear group transformation. This transformation leads to a highly nonlinear ordinary differential equation, which is then solved numerically using the Orthogonal Collocation Method.

Shukla H. S., Surati H. C., Timol M. G. [3], explores the three-dimensional magnetohydrodynamic nanofluid flow over a linearly stretching sheet, considering a non-Newtonian power-law model and convective boundary conditions. Using a deductive two-parameter group-theoretic similarity technique, the governing partial differential equations are transformed into ordinary differential equations, which are then solved numerically. The research examines the influence of key physical parameters, such as the power-law index, magnetic parameter, thermophoresis parameter, and Brownian motion parameter, on velocity, temperature, and concentration profiles.

Poonia M., Bhargava R. [4], investigated the laminar steady boundary layer flow of an incompressible MHD power-law fluid past a continuously moving surface. The research focuses on the influence of surface slip and non-uniform heat source/sink on flow and heat transfer.

The governing boundary layer flow equations are transformed into non-dimensional, non-linear coupled ordinary differential equations using similarity transformations. The Galerkin finite element method is employed to solve the resulting system. The study examines the impact of various physical parameters on velocity profiles, temperature distributions, and heat transfer rates for pseudoplastic and dilatant fluids.

Shafique M. [5], studied examines the diffusion-thermo (Dufour effect) in the MHD flow of a Jeffrey fluid past an exponentially accelerated vertical plate, incorporating chemical reaction and heat generation. The governing equations are transformed into non-dimensional forms and solved semi-analytically using the Laplace transform technique.

Khan W. A., Aziz A. [6], explores the MHD mixed convection flow near a moving horizontal plate, incorporating variable reactive index, magnetic field, and velocity slip. Using group theory, the governing equations are transformed into

similarity equations and solved numerically. The findings provide insights into heat and mass transfer rates under varying physical conditions.

Ece, M. C., Büyük, E. [7], investigated free convection heat transfer from a heated vertical plate to power-law fluids, focusing on similarity solutions. The governing equations are transformed into dimensionless forms using similarity variables and solved numerically. The findings provide insights into heat transfer characteristics for non-Newtonian fluids under varying thermal conditions.

Guedda M. [8], examined similarity solutions for boundary layer approximations in porous media, focusing on differential equations governing fluid flow. The analysis explores existence, non-uniqueness, and large-time behavior of solutions under specific conditions. The findings contribute to understanding free convection phenomena in porous environments.

Choubey K.R. [9], investigates the impulsive motion of a flat plate in a viscoelastic fluid under the influence of a transverse magnetic field. The analysis focuses on the fluid's response to sudden plate movement, considering magnetohydrodynamic effects and viscoelastic properties. The findings contribute to understanding boundary layer behavior in electrically conducting non-Newtonian fluids.

Kumari, M., Nath, G. [10], investigated the MHD boundary-layer flow of a non-Newtonian power-law fluid over a continuously moving surface with a parallel free stream. The governing equations are solved numerically using an implicit finite-difference scheme. The findings highlight the effects of magnetic parameters, Prandtl number, and fluid type on skin friction and heat transfer coefficients.

Mahmoud, M. A. [11], examined the effects of variable viscosity on hydromagnetic boundary layer flow along a continuously moving vertical plate in the presence of radiation. The analysis considers uniform suction and heat flux, exploring how viscosity variations influence velocity profiles, thermal boundary layer thickness, and skin friction. The findings contribute to understanding heat transfer mechanisms in electrically conducting fluids under magnetic fields.

Yadav D., Lee J. [12], investigates the combined effects of Hall current and magnetic field on the onset of convection in an electrically conducting nanofluid layer heated from below. Instead of prescribing a nanoparticle volume fraction on rigid impermeable boundaries, the authors assume a zero-nanoparticle flux condition, making the boundary condition more physically realistic.

Poonia and Bhargava [13], explored the heat transfer characteristics of a convective magnetohydrodynamic viscoelastic fluid flowing along a moving inclined plate with a power-law surface temperature. The authors employ the Finite Element Method to solve the governing equations, providing insights into how various physical parameters influence the fluid flow and thermal behavior. Their findings have applications in engineering and industrial processes where MHD effects play a crucial role in heat transfer.

Soh [14], investigated the unidirectional flow of an electrically charged power-law non-Newtonian fluid over a flat plate under the influence of a transverse magnetic field. The research focuses on deriving invariant solutions for the governing equations, providing insights into how electrical and magnetic effects interact with the fluid's non-Newtonian behavior. Timol M.G. [15], study examined the magnetohydrodynamic boundary layer flow of non-Newtonian power-law fluids near a suddenly accelerated flat plate. The research employs similarity transformations, including linear and spiral group transformations, to derive solutions for the governing equations. The findings indicate that the flow behavior aligns with established theoretical models, confirming the validity of the applied transformations.

PROBLEM FORMULATION

Consider the Magnetohydrodynamic (MHD) boundary layer flow of non-Newtonian power-law fluids near a flat plate.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial}{\partial y} \left(\left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) - \frac{g \sigma B_x^2(y, t)}{\rho} u \quad (1)$$

Where $\frac{g \sigma B_x^2(y, t)}{\rho}$ is magnetic field strength perpendicular to the plate we suppose it M_0 . u -velocity component along the plate (x -direction), y - coordinate normal to the plate, ν - kinematic viscosity, n - power-law index (for non-Newtonian fluid), σ - electrical conductivity of the fluid, ρ - fluid density

With the boundary condition as,

$$\begin{aligned} \text{For } y = 0, & \quad u = U(y, t) \\ \text{For } y = \infty, & \quad u = 0 \end{aligned} \quad (2)$$

Now introducing stream function $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, and the Magnetohydrodynamic boundary layer flow of non-Newtonian power-law fluids near a flat plate is as follow,

$$\frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial y} \right) = v \frac{\partial}{\partial y} \left(\left| \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y} \right) \right|^{n-1} \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial y} \right) \right) - \frac{g \sigma B_x^2(y, t)}{\rho} \frac{\partial \psi}{\partial y} \quad (3)$$

$$\text{With boundary conditions } y = 0, \quad \frac{\partial \psi}{\partial y} = U \text{ and } y = \infty, \quad \frac{\partial \psi}{\partial y} = 0 \quad (4)$$

METHOD OF SOLUTION OF A PROBLEM

To reduce a system of partial differential equation to ordinary differential equation, we often seek solutions with fewer independent variables. A common method is using a one-parameter group of transformations to exploit symmetries and simplify the system.

$$\begin{aligned} \bar{t} &= P^{k_1} t^*, & \bar{y} &= P^{k_2} y^* \\ \bar{\psi} &= P^{k_3} \psi^*, & \bar{S} &= P^{k_4} S^* \end{aligned} \quad (5)$$

Where k_1, k_2, k_3, k_4 and P are constants.

Deriving from equation (5) leads to the obtained result,

$$\left(\frac{\bar{t}}{t^*} \right)^{\frac{1}{k_1}} = \left(\frac{\bar{y}}{y^*} \right)^{\frac{1}{k_2}} = \left(\frac{\bar{\psi}}{\psi^*} \right)^{\frac{1}{k_3}} = \left(\frac{\bar{S}}{S^*} \right)^{\frac{1}{k_4}} = K \quad (6)$$

We get following invariants by solving one parameter group transformations as,

$$\eta = \frac{y}{t^{\alpha_2}}, \quad f(\eta) = \frac{u}{t^{\alpha_3}}, \quad \psi = C t^{\frac{\alpha_3}{\alpha_1}}, \quad S = \frac{S_0}{t^{\frac{\alpha_4}{\alpha_1}}}$$

FORMULATION OF ORDINARY DIFFERENTIAL EQUATIONS

Solving these invariants in equation (1) we get reduced ordinary differential equations as

$$v \frac{d}{d\eta} \left((f'(\eta))^{n-1} f'(\eta) \right) + \left(\frac{1 + (n-1)m}{n+1} \right) \eta f'(\eta) - m f(\eta) - S_0 f(\eta) = 0$$

With boundary conditions we have,

$$\begin{aligned} \text{For } \eta = 0, & \quad f(\eta) = C \\ \text{For } \eta = \infty, & \quad f(\eta) = 0 \end{aligned}$$

RESULTS AND DISCUSSION

Figure 1 shows the similarity velocity profile $f(\eta)$ for different values of the magnetic parameter S_0 in the magnetohydrodynamic (MHD) boundary-layer flow problem. The horizontal axis represents the similarity variable η , while the vertical axis represents the dimensionless velocity $f(\eta)$. In all cases, the velocity starts from its maximum value $f(0) = 1$ at the plate surface and decreases monotonically to zero as the distance from the plate increases, indicating that the fluid velocity gradually approaches the ambient stationary fluid. It is also observed that increasing the magnetic

parameter from $S_0 = 0$ to $S_0 = 1.0$ causes the velocity profiles to decay more rapidly. The curve corresponding to $S_0 = 1.0$ lies below those for $S_0 = 0.5$ and $S_0 = 0.0$, demonstrating that a stronger transverse magnetic field suppresses the fluid motion. This reduction in velocity is due to the Lorentz force generated by the interaction between the electrically conducting fluid and the applied magnetic field, which acts opposite to the direction of flow and resists the fluid motion. Consequently, higher values of the magnetic parameter produce a thinner momentum boundary layer and reduce the velocity throughout the flow field. Thus, the figure confirms that the applied magnetic field significantly controls the boundary-layer characteristics by damping the velocity distribution and enhancing the stability of the flow.

Figure 1: Similarity Velocity Profile $F(\eta)$

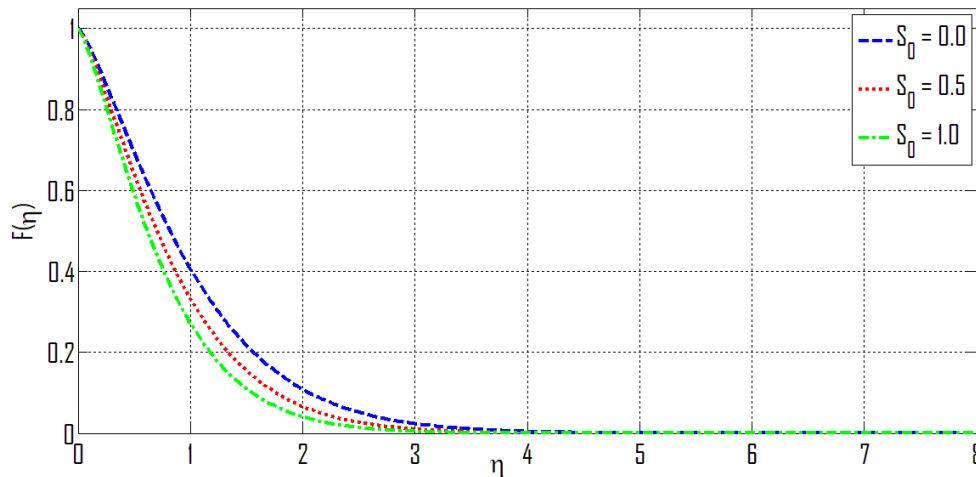
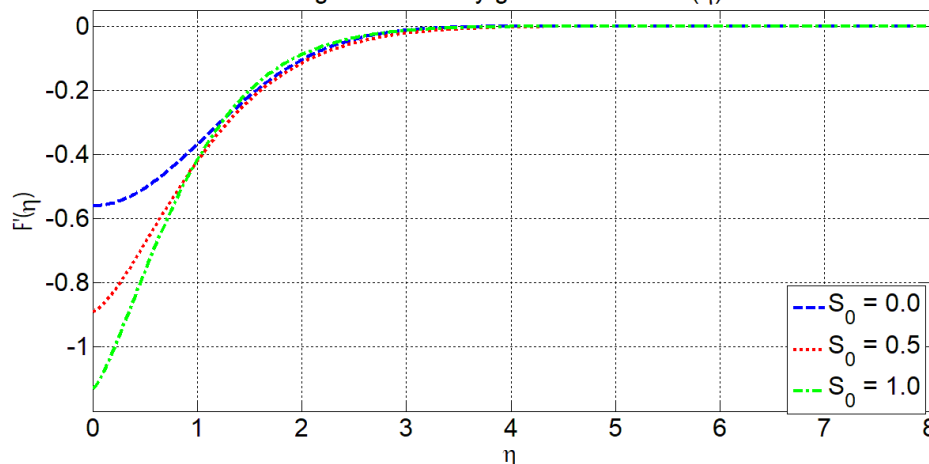


Figure 2 depicts the velocity-gradient profile $f'(\eta)$ for different values of the magnetic parameter S_0 in the magnetohydrodynamic (MHD) boundary-layer flow. The horizontal axis represents the similarity variable η , while the vertical axis represents the dimensionless velocity gradient $f'(\eta)$ which is directly related to the wall shear stress or skin-friction coefficient. Near the plate surface ($\eta = 0$), the velocity gradient is negative because the fluid velocity decreases away from the moving plate. As the similarity variable increases, all the profiles gradually approach zero, indicating that the velocity gradient vanishes outside the boundary layer where the fluid reaches the quiescent state. It is evident that increasing the magnetic parameter from $S_0 = 0$ to $S_0 = 1.0$ makes the initial velocity gradient more negative. This behavior indicates that a stronger transverse magnetic field increases the magnitude of the wall shear stress due to the enhanced **Lorentz force**, which opposes the fluid motion and produces greater resistance to the flow. Consequently, the momentum boundary layer becomes thinner, causing a steeper velocity gradient near the wall. Far from the plate, the influence of the magnetic field gradually diminishes, and all the curves converge smoothly toward zero. Therefore, the figure demonstrates that the magnetic parameter significantly influences the shear characteristics of the boundary layer by increasing the surface drag while simultaneously suppressing the fluid velocity, which is consistent with the physical behavior of electrically conducting fluids under an applied magnetic field.

Figure 2: Velocity-gradient Profile $F'(\eta)$



CONCLUSION

In this work, the effects of the magnetic parameter on the MHD boundary-layer flow characteristics were investigated through similarity solutions. The numerical results showed that the presence of a transverse magnetic field suppresses the fluid motion by generating a Lorentz force opposite to the direction of flow. As the magnetic parameter increases, the dimensionless velocity decreases throughout the boundary layer, leading to a reduction in momentum boundary-layer thickness. At the same time, the magnitude of the velocity gradient near the wall increases, resulting in higher wall shear stress and skin-friction coefficients. These findings indicate that the magnetic field plays an important role in controlling the transport characteristics of electrically conducting fluids. Therefore, magnetic fields can be effectively utilized in engineering and industrial applications involving MHD flow systems, such as cooling technologies, metallurgical processes, and electromagnetic flow control devices.

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