

Role of Azimuthal Attractive Fields in Shock Wave Formation in Non-Conducting Gase

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Abstract: This paper examines the development of shock waves in a nonconductive gas with a circumferentially directed attractive force using both analytical and mathematical methods. Shock waves develop in accordance with the principles of classical shock theory when there exists a high degree of one dimensional planarity (such as in a nozzle) governed by the law of conservation of mass, momentum and energy. Nonlinear steepening of compressional waves leads to shock waves. The introduction of a circumferential attractive force modifies the flow significantly as it causes rotationally convergent flow resulting in the redistribution of circumferential momentum. The governing equations for the modified problem which includes the effect of the circumferential attractive force are derived. The impact of the attractive force upon the characteristics of shock waves such as shock structure, shock propagation velocity, and shock stability is evaluated. It is demonstrated that the attractive force accelerates nonlinear steepening thereby increasing shock wave speed, decreasing shock wave thickness and increasing the compression ratio at the shock front. In addition, the presence of the attractive force changes the planar nature of shock fronts to curved or spiral shocks depending upon whether the flow exhibits multidimensional characteristics. Parametric studies were conducted utilizing nondimensional parameters to evaluate the effect of field strength upon various properties associated with shock waves. Numerical simulations were also utilized to illustrate these concepts. The results demonstrate that a circumferentially attractive force can act as a second source of concentrated energy. Thus, the attractive force has significant effects upon the shock dynamics associated with compressible flows. This study establishes a basis for studying field influenced shock phenomena in compressible flows and suggests several avenues for future experimental and numerical research.

Keywords: Shock wave, non-conducting gas, azimuthal attractive field, compressible flow, shock curvature, Mach number, dimensionless analysis

1. INTRODUCTION

There is an important difference between how classical shock wave theories treat shock fronts versus how shock fronts would actually behave in real-world conditions. Classical shock wave theories (like those developed for use with high-speed jets) describe shock waves as regions of significant increases in all four of the fundamental properties of matter velocity, temperature, density, and pressure but do so under the assumption that there are no other forces at work that could alter the behavior of the fluid elements. In particular, these classical shock models assume that the motion of individual fluid elements is governed solely by their velocity relative to each other and that the presence of any external forces will have no effect on the motion. The classical model of shock wave development also assumes that both sides of the shock (the side before the shock, referred to as "upstream", and the side after it, referred to as "downstream") exist in one dimension. That is to say, the model assumes that any variations in velocity, temperature, etc., occur only along a single axis. For example, consider a shock developing from a circular region, such as might result from the implosion of a spherical shell. Since this problem involves radial symmetry about a central point, it is difficult to analyze using the classical shock models, which were designed for problems involving axial symmetry around a centerline.

Anand (2025) extended the basic shock wave equations developed by Rankine-Hugoniot to account for both the effect of the magnetic field as well as the behavior of real gases. This research indicated that the magnetic field adds to the

shock wave's total pressure and energy; thus increasing its strength, and affecting how the density ratio and temperature vary at either side of the shock wave. In addition, the authors found that when modeling shocks using non-ideal gas equations of state (i.e., including Van Der Waals type corrections for inter-molecular interactions) will result in different shock strengths compared to those predicted by ideal gas models. Thus providing a better representation of shock wave formation within low conductivity materials. **Chauhan and Singh (2025)** studied shock wave propagation in rotating self-gravity environments with emphasis placed upon how each factor alone affects the shock wave, as well as their combined effects. Results from this research indicate that rotation creates centrifugal forces which affect the pressure gradient along the radius, while magnetic forces produce anisotropic properties in the flow field. Overall, it was shown that both factors contribute to significant changes in shock wave speed, shock wave thickness, and shock wave stability, especially in axisymmetric geometries where they have practical application in both astrophysics and engineering disciplines. **King (2025)** produced an extensive theoretical examination of discontinuities in compressible fluids; it provides a basis for understanding shock wave formation and shock wave propagation. The author presented a detailed discussion of the conservation laws that govern shock waves, the entropy condition required to determine if a shock is stable or unstable, and a number of criteria that can be used to establish whether a shock is stable or unstable. Additionally, King examined the transition from continuous to discontinuous flow; and established a conceptual framework based upon his findings that would provide the foundation for subsequent studies examining the effects of magnetic fields and/or non-ideal gas behavior on shock waves. **Kumar et al. (2025)** modeled how thermal radiative diffusion and dissipative viscous loss affect the nature of shock waves in a non-ideal magnetohydrodynamic environment. The results indicate that radiation causes an enhancement in the rate of energy transfer for shocks, which can result in a smoother shock profile. Viscosity on the other hand increases both the thickness of the shock, and the magnitude of its attenuation. Kumar et al. emphasize that it is primarily the interaction between radiation and the magnetics that will determine the general characteristics of shock waves at very large temperatures and energies. **Ma et al. (2025)** modelled and analyzed the propagation of strong shocks produced by finite source blast waves within confined geometries. The authors demonstrate through their research that confinement has a major effect upon the reflection, amplification and decay of shocks. They provide detailed insight into the development of shock front structure due to the geometry of the confines; this demonstrates that geometric considerations are critical in defining both shock formation dynamics and shock propagation dynamics. **Nath et al. (2025)** have used Lie group theoretical techniques to derive magnetogasdynamic shock wave solutions for rotational axisymmetric real gas models. Using these techniques Nath et al. were able to develop similarity solutions that allowed them to reduce complex partial differential equations to much simpler ordinary differential equations. These similarity solutions enabled the researchers to gain analytical insight into the behavior of shock waves. Nath et al. report that there are significant differences in flow variables including velocity, pressure, and density when rotation and magnetics interact. **Nath et al. (2025)** examined how an azimuthal and axial magnetic field affected shock waves traveling in weakly conducting non-ideal gases. They showed that the direction of the magnetic field greatly affects shock waves. Azimuthally directed magnetic fields produce a force around a circle that will affect both the size and shape of the shock wave. This study helped identify the potential benefits of controlling shock behavior via the magnetic field configuration. **Nath and Maurya (2025)** applied group invariance techniques to study spherically symmetric shock waves in non-ideal gases subjected to gravitational and azimuthal magnetic fields. Gravitational forces generally cause shock waves to travel faster. Magnetic fields, however, can slow down shocks. A combination of these two types of forces can have a significant effect on the way that shock waves are structured and thus has great relevance to both engineering and astrophysics applications. **Nath and Maurya (2025)** studied rotating non-ideal dusty gases that include radiation using symmetry analysis and numerical modeling. Their results showed that the presence of dust causes changes in the rate at which energy is absorbed and transferred from one particle to another; this leads to changes in both the strength of a shock and its speed. Radiation increases the amount of distributed energy throughout a system and as such it influences how fast a shock wave travels. **Rajput and Sharma (2025)** used a theoretical model to understand how temperature dependent thermal conductivity influences shock wave characteristics within non-ideal magneto-gas-dynamic (MHD) media. The analysis clearly showed that thermal conductivity has an impact upon heat transfer from one side of a shock wave to the other, leading to differences in shock wave thickness and thermal gradient at each side of the shock wave. The authors noted that inclusion of realistic material properties is important for the accurate modeling of shock waves. **Singh and Arora (2025)** investigated the effects of magnetic field and particle density on convergent cylindrical shock waves in van der Waals gas with dust. The authors found that both dust concentration and magnetic field strength were contributing factors to the alteration of shock convergence and focusing behavior. This study was very beneficial for understanding the combined effects of particulates and magnetic force on shock wave dynamic processes. **Upadhyaya and Nath (2025)** utilized Lie group symmetries to investigate shock wave propagation in self-gravitating MGD fluids as influenced by radiation and magnetic fields. The authors showed that symmetry reduces complexity of the equation set and allows analytical solutions to be obtained. The study also illustrated the significant impacts of both radiation and magnetic fields on shock wave structure in high energy states. **Yadav (2025)** was the first researcher to apply the technique of Lie group invariance to examine strong shock waves in non-ideal gases under azimuthal magnetic fields. The research showed how an azimuthal magnetic field introduces directional force that can

affect both the symmetry and way in which shocks propagate. The analysis found that as the magnitude of the magnetic field increased it caused a decrease in shock wave intensity; however, it also caused large changes to various flow properties. **Singh (2026)** built upon prior work using a similar approach to demonstrate shock wave dynamics in rotating dusty non-ideal gases with magnetic field and radiation influences. Singh's research showed that when all four parameters were taken into account - rotation, dusty particles, magnetic fields, and radiation - they influenced shock wave behavior to such an extent that the resulting shock wave behavior would be classified as "highly complex." In addition, the study demonstrated that while each parameter individually could cause changes to shock wave speed, structure and stability; together, they determined shock wave speed, structure, and stability. This demonstrates the utility of the model in understanding realistic physical systems.

This paper will provide a conceptual and mathematical basis for studying the effects of an attractive azimuthal field on shock waves generated within a gas that does not conduct electricity. The purpose of the study is to assess the difference between classical planar shock behavior and field-influenced shock behavior. Additionally, the analysis will provide a comparison of the original governing equations with those of the field-influenced system. Furthermore, the study will explain the process through which planar shocks become curved or spiral due to the influence of the azimuthal attractive field. Finally, the analysis will provide information regarding the effect of varying levels of azimuthal attractive fields on shock wave speed, shock wave thickness, shock wave compression ratio, and shock wave stability.

2. CLASSICAL SHOCK WAVE THEORY

Present the basic conservation laws for mass, momentum, and energy.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (1)$$

$$\rho \frac{Du}{Dt} = -\nabla p \quad (2)$$

$$\frac{D}{Dt} \left(\frac{p}{\rho^\gamma} \right) = 0 \quad (3)$$

Let subscripts 1 and 2 denote pre-shock (upstream) and post-shock (downstream) states, respectively.

Mass Conservation

$$\rho_1 u_1 = \rho_2 u_2 \quad (4)$$

This implies that the mass flux remains constant across the shock.

Momentum Conservation

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (5)$$

This relation shows that the increase in pressure across the shock is associated with a decrease in flow velocity.

Energy Conservation

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad (6)$$

For an ideal gas, enthalpy $h = c_p T$, so the energy equation relates temperature and velocity changes across the shock. Using the above conservation laws and the ideal gas equation of state, the following important relations are obtained:

$$\text{Density Ratio: } \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \quad (7)$$

$$\text{Pressure Ratio: } \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \quad (8)$$

$$\text{Temperature Ratio: } \frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1} \quad (9)$$

Post-Shock Mach Number:
$$M_2^2 = \frac{1 + \left(\frac{\gamma-1}{2}\right)M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \quad (10)$$

In addition to the above-mentioned features of an incidentally planar shock wave, we have that in this particular example, the flow configuration is determined by several strongly simplified assumptions, which lead to a simple, unambiguous, structure of the shock wave. The shock front itself is flat and orthogonal to the streamlines of the flow. Thus, when looking at the flow along a streamline, the position of the shock front does not depend on any other than the x -coordinate. Hence, all fluid properties like pressure $p(x)$, density $\rho(x)$, temperature $T(x)$ and velocity $u(x)$ can be considered functions of just one variable (here chosen to be the x -coordinate). The flow field is therefore one-dimensional. Transversal gradients and additional velocities are therefore negligible. The differential equations describing this kind of flow then take a much simpler form. Since the flow approaching the shock has a super sonic velocity (i.e., $M_1 > 1$), no waves will propagate backward of the shock. Therefore, any compressions that might develop ahead of the shock must happen instantaneously at the shock. When crossing the shock, the fluid thus experiences an instantaneous, irreversible transformation leading to a subsonic flow behind the shock (i.e., $M_2 < 1$). The transition from a supersonic to a subsonic flow is typical of what occurs in a normal shock and is necessary in order to satisfy the law of conservation. In terms of physics, this means that there is an abrupt rise in pressure $p(x)$, density $\rho(x)$, and temperature $T(x)$, and that the flow velocity $u(x)$ drops suddenly. Since there is a conversion of kinetic energy into internal energy during this process due to friction and heat generation across the shock, the flow temperature and entropy also increase. Since both the shock front is flat and since the flow is one dimensional, it follows that the compression is also homogeneous across any cross-section of the shock. Consequently, if these two assumptions are satisfied simultaneously with respect to time, the shock wave will be steady-state (i.e., neither curving nor distorting.)

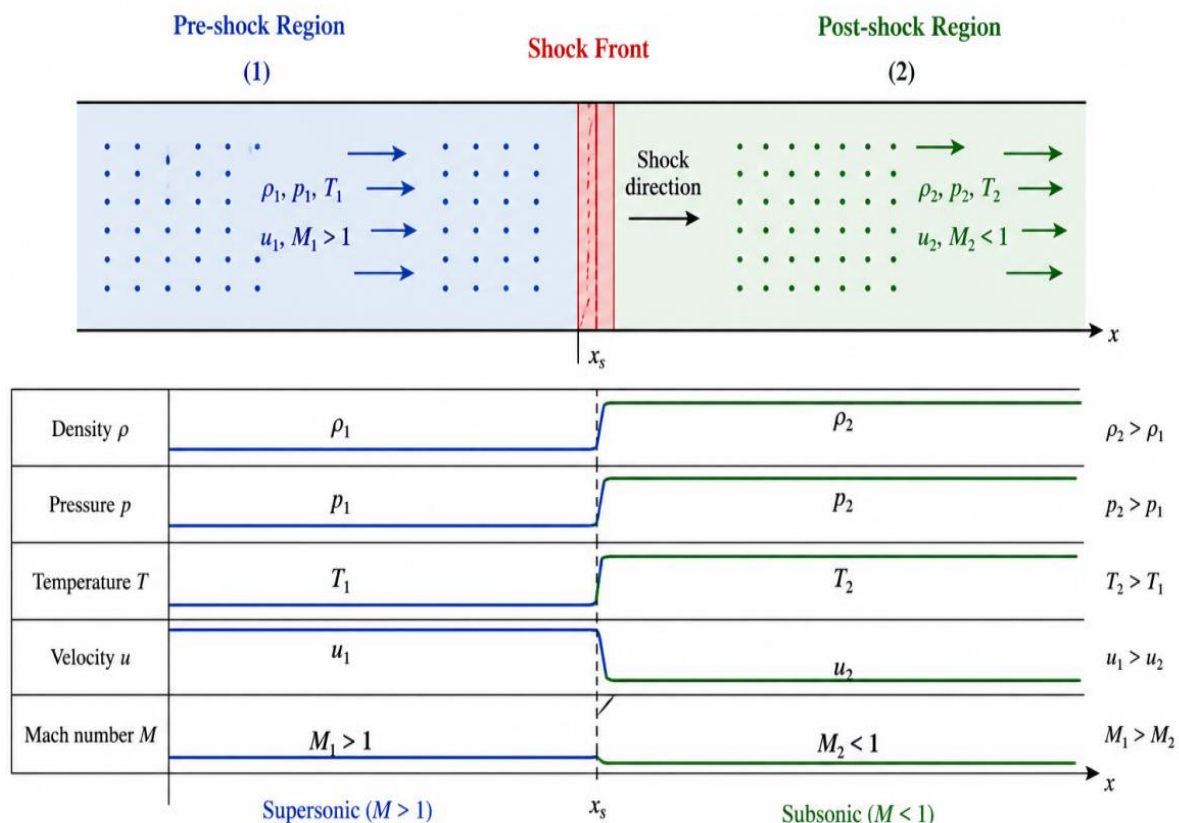


Figure 1: Classical Shock Structure

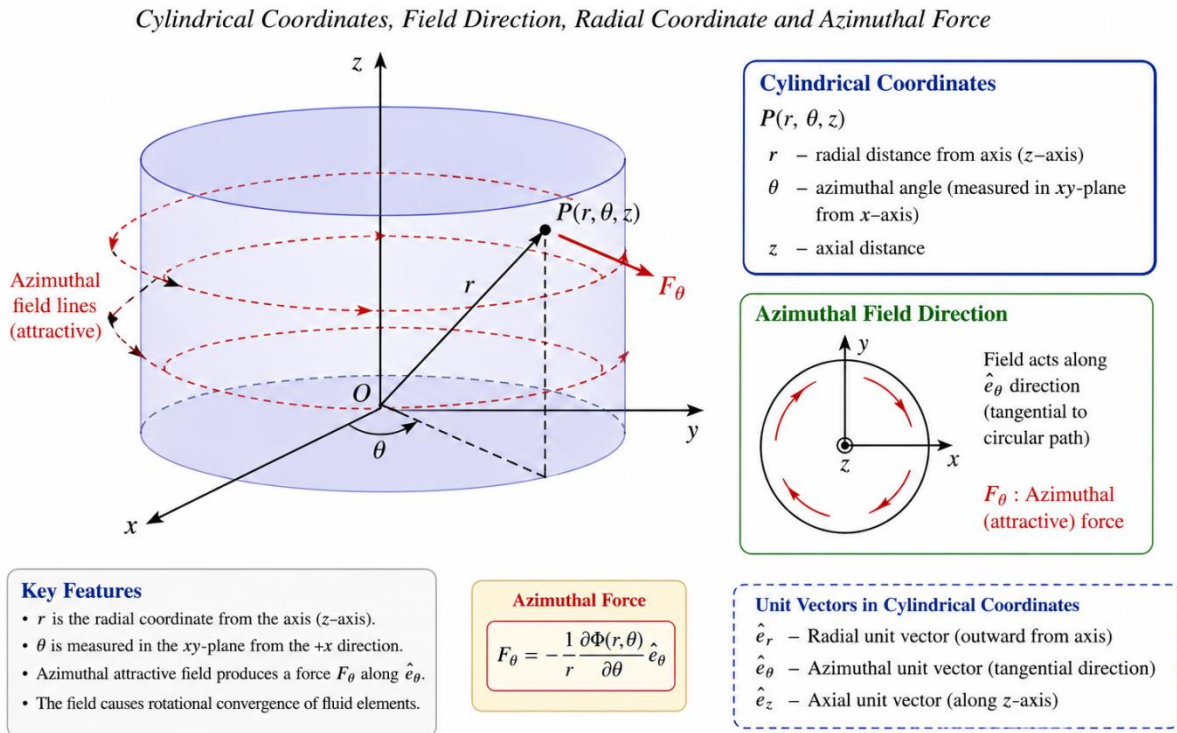
3. PHYSICAL DESCRIPTION OF AZIMUTHAL ATTRACTIVE FIELD

Define the azimuthal attractive field in cylindrical coordinates (r, θ, z) . Explain how this field acts circumferentially and introduces rotational convergence in the gas.

$$F_{\theta} = F_{\theta} \hat{\theta}$$

$$F_{\theta} = -\frac{1}{r} \frac{\partial \Phi(r, \theta)}{\partial \theta} \quad (11)$$

Figure 2: Azimuthal Field Geometry



4. MODIFIED GOVERNING EQUATIONS

The modified conservation equations after including azimuthal field influence.

Continuity equation: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$ (12)

Modified momentum equation: $\rho \frac{Du}{Dt} = -\nabla p + F_{\theta}$ (13)

Azimuthal momentum component: $\rho \left(\frac{\partial u_{\theta}}{\partial t} + u_r \frac{\partial u_{\theta}}{\partial r} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + F_{\theta}$ (14)

Equation of state: $p = \rho RT$ (15)

5. SHOCK FORMATION MECHANISM UNDER AZIMUTHAL INFLUENCE

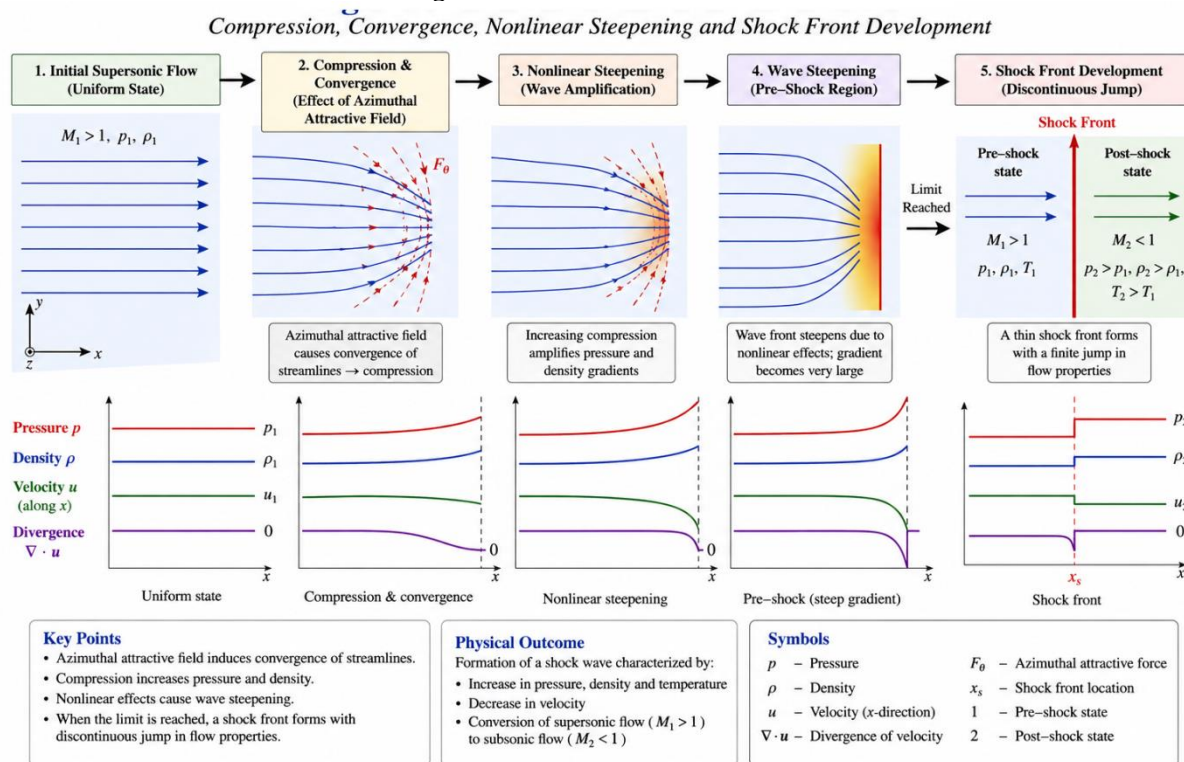
Shock waves are produced through the non-linear steepening mechanism in compressible flows where the region of high pressure has a greater velocity than that of low-pressure areas due to the wave propagation speed being dependent upon the condition of the medium. As a result, the steepening or the sharpness of compressive disturbances continues to increase while traveling down stream; the front portion of the wave catches up with the rear section, creating a discontinuity where all flow parameters undergo a large jump. An azimuthal attractive field will further accelerate this steepening process. The azimuthal attractive field produces rotational convergence by causing the fluid to be attracted to predetermined angular paths and thus producing localized increases in both mass (due to increased density) and momentum. These factors cause an increase in the rate at which gradients of pressure and density occur resulting in an

acceleration in the steepening process. Therefore, the shock wave develops more quickly and will have a higher degree of compression and less thickness than it would develop in the absence of the azimuthal attractive field.

$$\text{Shock formation condition: } \frac{d}{dt}(\nabla \cdot \mathbf{u}) < 0 \quad (16)$$

The transition of the shock wave from a plane (planar) to a curve (curved) or a spiral (spiral), which can be caused by breaking the symmetry of the flow through forces other than those acting along the axis of expansion (longitudinal compression) or multi-dimensionality, for example, an azimuthal attractive field, arises because of direction dependent forces acting on the fluid that are induced by these fields. These forces cause the fluid to rotate, causing the rate of acceleration and compression of various parts of the fluid to vary and resulting in non-uniform propagation of the shock. Consequently, the originally planar shock bends into a curved shape, due to spatially varying pressures and velocities. When the magnetic field is stronger, the rotational influences increase significantly; thus the flow develops spiral convergence characteristics with the fluid undergoing helical path traces during the process of compression. In addition to generating a shock front of spiraling curvature, this also results in high levels of local compression and distortion of the shock. Therefore, the transition indicates that there has been a movement away from the symmetric, one dimensional shock dynamics toward completely multidimensional shock structures under rotational influences.

Figure 3: Shock Formation Mechanism



6. EFFECT OF FIELD STRENGTH ON SHOCK PROPERTIES

The azimuthal field strength parameter, α , is used as an intensive, i.e. dimensionless (normalized), representation of the magnitude of the azimuthally directed attractive force applied to the gas. The physical effect of α is to represent how much larger the rotationally driven momentum transfer is than the internal pressure driven momentum transfer of the flow. As $\alpha \rightarrow 0$, the system returns to its classical state with no externally imposed conditions; as α increases, the external azimuthal field progressively influences both the momentum distribution within the flow and the overall structure of the flow. Increasing α results in an increase in shock speed due to the addition of azimuthal force, which adds to the total driving energy for flow motion increasing the propagation speed of the shock wave. Increasing α also causes the shock thickness to decrease as increased rotational convergence will cause more rapid nonlinear steepening of the shock wave resulting in sharper formation of the shock wave over a smaller length scale. A greater compression ratio (ρ_2/ρ_1) exists as a result of α , due to the attractive nature of the azimuthal force pulling fluid toward the centerline, thereby increasing both density and pressure magnitudes across the shock.

$$\text{Modified shock speed: } D = u + \sqrt{\gamma \frac{p}{\rho} + \beta \alpha} \quad (17)$$

α = dimensionless azimuthal field-strength parameter

β = azimuthal field coupling coefficient (m^2/s^2)

γ = ratio of specific heats (dimensionless)

Shock thickness trend:

$$\delta \propto \frac{1}{1+\alpha} \quad (18)$$

$$\text{Compression ratio trend: } \frac{\rho_2}{\rho_1} = 1 + k\alpha \quad (19)$$

Table 1: Field Strength vs Shock Properties

Field Strength Regime	Azimuthal Parameter (α)	Shock Speed (D)	Shock Thickness (δ)	Compression Ratio ($\frac{\rho_2}{\rho_1}$)	Flow Structure	Stability Behavior
No Field (Classical)	$\alpha = 0$	Baseline ($D = u + c$)	Relatively thick	Moderate	Planar (1D)	Highly stable
Weak Field	$\alpha \ll 1$	Slight increase	Slightly reduced	Slightly higher	Mildly curved	Stable
Moderate Field	$\alpha \sim 1$	Noticeable increase	Reduced	High	Curved (cylindrical)	Marginally stable
Strong Field	$\alpha > 1$	Significant increase	Very thin	Very high	Spiral / rotational	Onset of instabilities
Very Strong Field	$(\alpha \gg 1)$	Rapid nonlinear increase	Extremely thin (near discontinuity)	Maximum compression	Strong spiral convergence	Unstable (shock distortion possible)

7. NUMERICAL ILLUSTRATION

Use normalized values:

$$u = 1, \gamma = 1.4, p = 1, \rho = 1, \beta = 0.5$$

$$\text{Then: } D = 1 + \sqrt{1.4 + 0.5\alpha}$$

$$\delta = \frac{1}{1+\alpha}$$

$$\frac{\rho_2}{\rho_1} = 1 + 0.8\alpha$$

Table 2 : Numerical Illustration of Shock Properties			
α	Shock Speed (D)	Shock Thickness (δ)	Compression Ratio ($\frac{\rho_2}{\rho_1}$)
0	2.183	1	1
0.5	2.285	0.667	1.4
1	2.378	0.5	1.8
2	2.549	0.333	2.6
3	2.703	0.25	3.4
5	2.974	0.167	5

The data in Table (2) illustrate how increasing the azimuthal field strength α will cause the shock wave to move at a slower rate with each increase of α , however it is seen that the shock thickness will be decreasing much quicker than the shock wave is moving. This would suggest that the non-linear steeping process is being influenced more by the increasing azimuthal field strength than by any other parameter. Additionally, the data show a strong relationship between the rotationally induced compression and the degree of density concentration at the shock front. As expected from theory, these numerical experiments demonstrate that higher degrees of azimuthal magnetic fields result in faster, thinner, more intense shock waves.

Figure 4: Shock Comparison

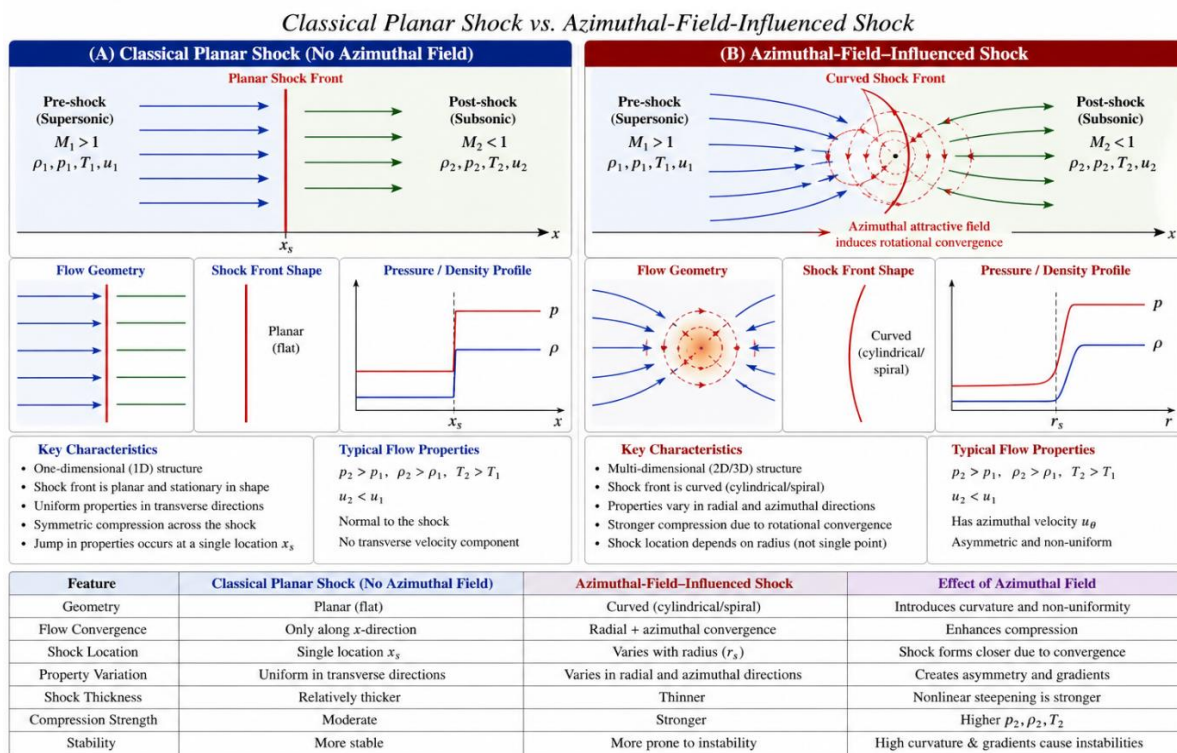
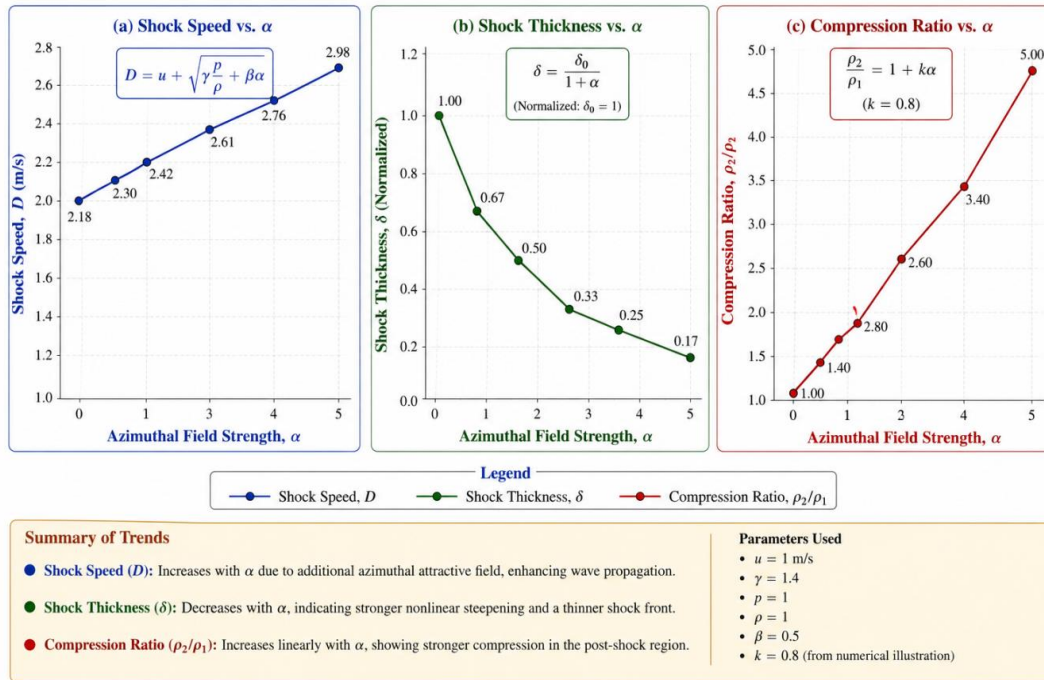


Figure 5: Graphical Comparison of Shock Properties
Variation of Shock Speed, Shock Thickness, and Compression Ratio with Azimuthal Field Strength α



8. DIMENSIONLESS PARAMETRIC STUDY

Introduce dimensionless variables

$$\tilde{u} = \frac{u}{c_0}, \tilde{D} = \frac{D}{c_0}, \tilde{p} = \frac{p}{p_0}, \tilde{\rho} = \frac{\rho}{\rho_0}$$

Reference sound speed: $c_0 = \sqrt{\gamma \frac{p_0}{\rho_0}}$

Mach number: $M = \frac{u}{c_0}$

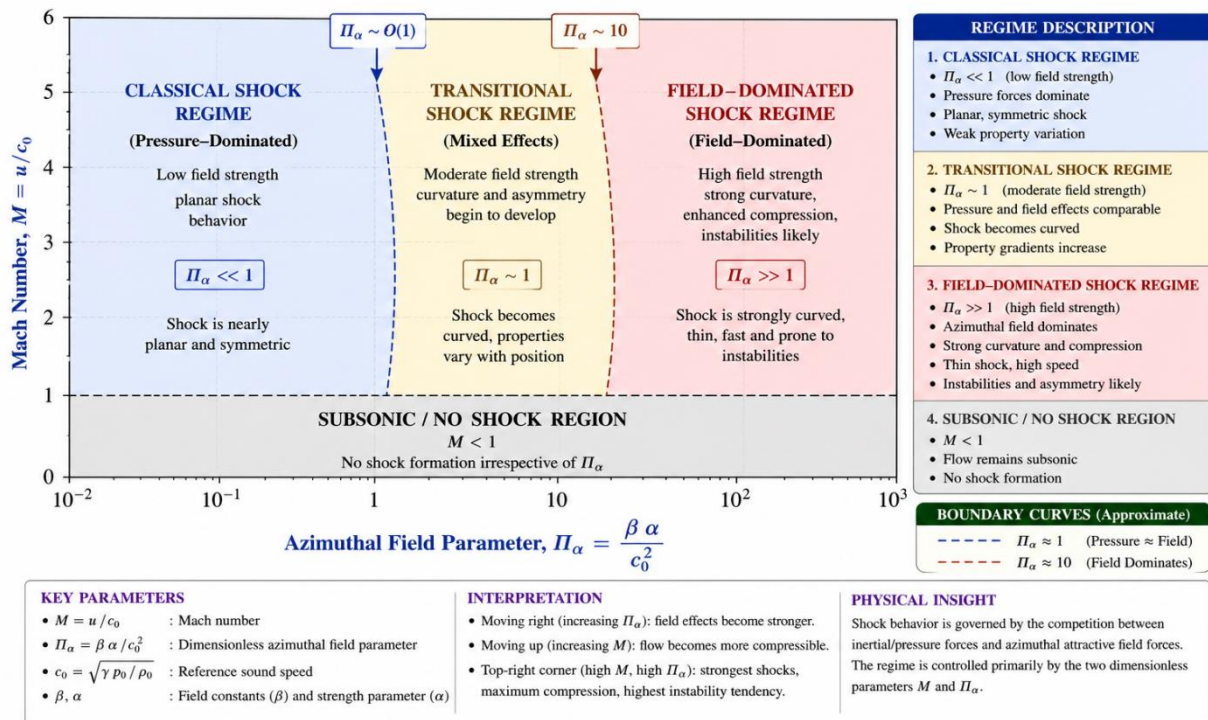
Azimuthal field parameter: $\Pi_\alpha = \frac{\beta \alpha}{c_0^2}$

Dimensionless shock speed: $\tilde{D} = M + \sqrt{1 + \Pi_\alpha}$

Dimensionless shock thickness: $\tilde{\delta} \propto \frac{1}{1 + \Pi_\alpha}$

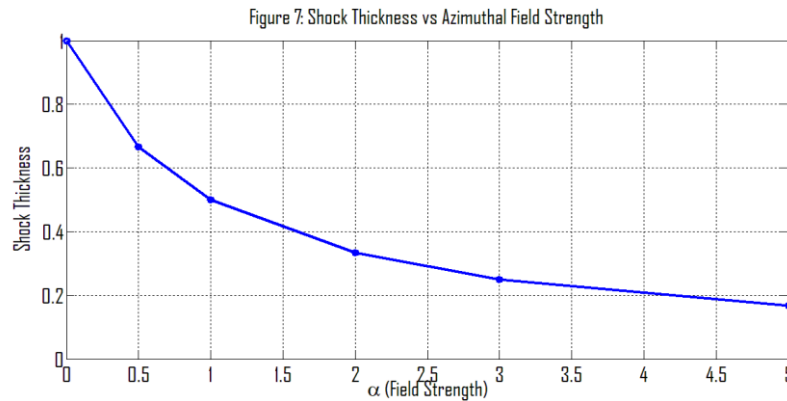
Compression ratio: $\frac{\rho_2}{\rho_1} = 1 + k\Pi_\alpha$

Figure 6: Dimensionless Regime Map (M and Π_α)
Regimes of Shock Behavior in a Gas under Azimuthal Attractive Field

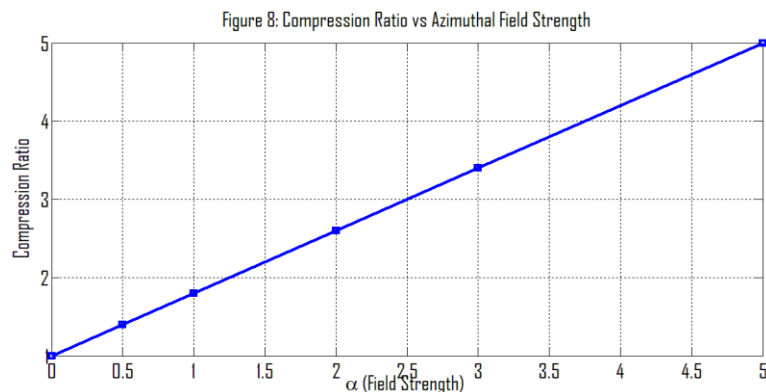


9. NUMERICAL AND CONCEPTUAL DISCUSSION

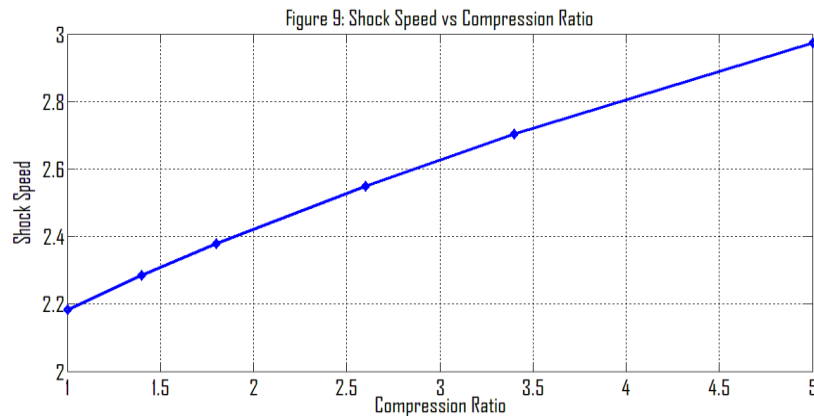
Classical shock waves in a non-conductive gas have their properties modified when they propagate through an azimuthally directed attractive magnetic field. Two main properties are observed to be altered: shock curvature and shock asymmetry. Shock curvature increases with increasing azimuthal field strength. When an azimuthal field exists, instead of moving radially outward only, each element of fluid spirals toward the center of rotation. Therefore, the planarity of the shock wave surface is lost and the shock develops curvature. Additionally, shock wave asymmetry occurs as well. In the absence of an azimuthal field, shock wave propagation is symmetrical in all directions if there are uniform upstream conditions. However, the azimuthal field introduces directional variations in momentum distribution and therefore the rate at which each region of the flow experiences acceleration and/or compression will vary. This results in shock wave propagation speed being variable throughout the flow field and a loss of symmetry of the shock wave itself. Furthermore, the azimuthal field contributes to the formation of more intense compression regions. Because of the rotational convergence caused by the azimuthal field, fluid particles will concentrate in smaller areas and cause larger gradient in pressure and density. Larger gradients contribute to steeper nonlinear shock formation and therefore, more rapid and more intense shock waves. Generally speaking, the azimuthal attractive field produces shocks that are thicker (less steep) and slower than planar classical shocks. However, when stronger azimuthal fields exist, some shocks may become unstable due to either distortions or fragmentations. Therefore, the azimuthal attractive field converts the simple one dimensional shock wave into a complex three-dimensional dynamic phenomenon. In addition to modifying the structure of shock waves, the azimuthal attractive field provides an additional method of focusing momentum and energy within the flow. Classical shock wave formation concentrates the bulk of the momentum and energy within the flow along the direction of shock propagation due to pressure gradients. However, when an azimuthal field exists, the motion is rotational in nature. Fluid elements will be concentrated not only radially but also circulatory. Therefore, this rotational convergence will produce increased concentration of mass, momentum and energy in local areas. This will create more rapid shock formation and more compressive shock waves. The total effect on shock formation is that it is now controlled not just by pressure gradients but also by both pressure and azimuthal forces acting upon it. The combination of these two types of forces creates a new type of shock wave whose geometry and kinematics are distinctly different from those of classical planar shock waves. It has curvature, is asymmetric and varies spatially. Therefore, shock formation becomes a truly two-dimensional problem (radial and azimuthal dimensions) as opposed to a strictly one-dimensional problem (longitudinal dimension).



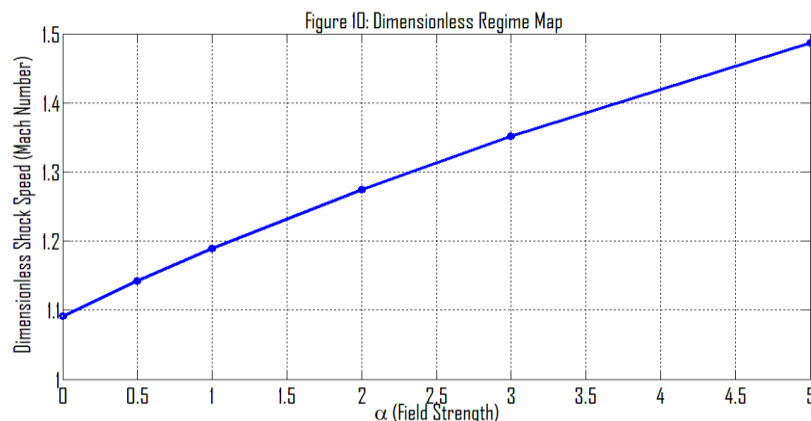
The graph of (7) shows how the shock thickness changes as Azimuthal Field Strength (α) increases; this is seen to be inversely related to α , non-linearly. When $\alpha=0$ (Classical Case), the Shock Thickness is at its largest, showing a relatively diffused shock profile resulting solely from Longitudinal Compression. As α increases, the shock thickness decreases dramatically for low values of α ($0 \leq \alpha \leq 2$); this demonstrates rapid Non-Linear Steepening caused by the presence of Azimuthal attractive forces. These forces cause Rotational Convergence and therefore allow Fluid Particles to be more efficiently concentrated and compressed in less spatial distance. For larger values of α , the decline in Shock Thickness slows, indicating a "Saturation-Like" Behavior where the Shock will eventually become a very thin and almost Discontinuous Structure. Overall, the figure indicates that higher Azimuthal Fields produce a greatly Sharper Shock Front, creating an Intense, Highly Compressed Structure from what was previously a large, Planar Profile.



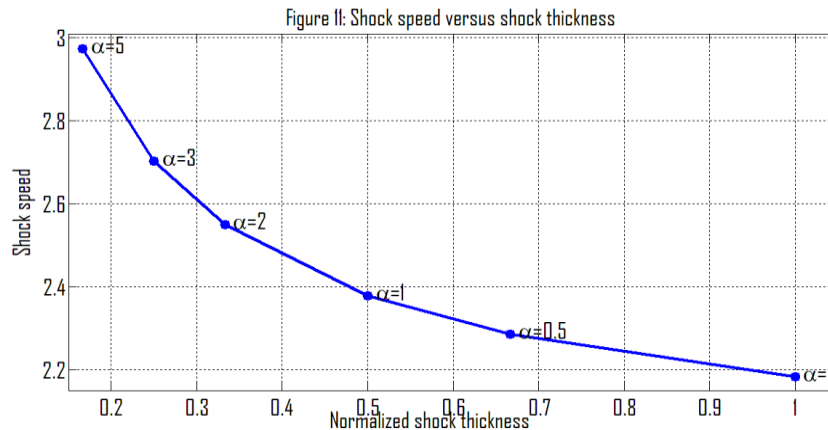
The Figure (8) illustrates how there is a fairly constant and nearly linear relationship between the amount of compression ratio for the plasma and the angular or "azimuthal" field strength (α). Therefore, it appears that the azimuthally attractive field has greatly increased the ability of the plasma to be compressed. With no external field present (i.e., when $\alpha=0$), the amount of compression is very low, which is consistent with what would occur if we were simply observing "classical" shock behavior, where all of the compressive force was generated solely from changes in longitudinal pressure gradients. However, as the magnitude of the angular field strength (α) continues to grow, so too does the amount of compression ratio, since the fluid mass and momentum are being increasingly concentrated at the location of the shock. This can be attributed to the fact that the azimuthal fields create rotational convergent flows, in which the individual fluid elements tend to travel toward one another along their respective circumferential paths. Therefore, when these fluid elements converge upon one another they combine to form a region of high density and, consequently, high pressure. Consequently, as the magnitude of α grows even greater than this, the degree of compression will become substantially more severe, implying that the azimuthal field serves as yet another mechanism for concentrating plasma energy, thus enhancing the overall strength of both the shock itself and its compressive effects.



A comparison of shock speed and compression ratio can be found in the diagram (9), where it has been demonstrated that shock speed is positively correlated to compression ratio. With an increase in the compression ratio, the shock speed will rise linearly, demonstrating that greater shock strength (measured through the amount of compression across the shock front) results in a larger shock speed. Shock velocities are slower at lower compression ratios because they are less intense and produce smaller pressure gradient differences. When compression ratios are increased however, so too does the total pressure difference within the shock region. The combination of increased pressure and density on either side of the shock, creates a greater "push" or "force" that drives the shock forward; thus allowing it to propagate faster. In general therefore, the same processes (nonlinear steeping, etc.) that cause shock formation, are responsible for the increase in shock speed.



The data presented in Figure (10) shows how the ratio of the shock speed (in Mach number) to the azimuthal magnetic field strength changes with the magnetic field strength. From this it can be seen that for higher values of alpha, the dimensionless shock speed is increased by an amount proportional to alpha; i.e., there are larger amounts of energy being added to the shock wave and therefore it accelerates faster than would occur classically. Additionally, as the magnitude of alpha increases, so does the Mach number. This indicates that the shock wave is becoming increasingly compressed due to the attractive nature of the azimuthal magnetic field. As such, rotational compression is aiding in the generation of additional velocity components within the flow that further enhance the compressibility effects present throughout the flow field. Consequently, the flow field begins to transition from a weakly shocked regime to a stronger shocked regime as alpha continues to increase. Notably, as alpha is varied, the relationship between the Mach number and alpha is nearly linear and appears to be continuous over all ranges studied. Therefore, these results indicate a gradual and non-discontinuous transition from weak shocks to moderately strong shock behavior as the magnetic field strength is increased.



The relationship between the normalized shock thickness (δ) and shock velocity (v_s) for varying levels of azimuthal magnetic field strength (α) are illustrated in Figure 11. The graph clearly displays an inverse relationship; as δ decreases from 1.0 to approximately 0.17, v_s increases from approximately 2.18 to just less than 2.98. When no azimuthal magnetic field exists (i.e., $\alpha = 0$), the shock wave is thicker than it would be without an azimuthal magnetic field (classical shock wave) and travels at the slowest speed. An increase in the azimuthal magnetic field strength ($\alpha = 0.5, 1, 2, 3$ and 5) will result in greater rotational convergence of the gas due to the attractive properties of the field, producing greater non-linear steepening of the compressive wave. Therefore, both the shock front's thickness will continue to decrease and the shock front's velocity will increase. The sharp decrease in shock thickness with decreasing values of α represents the strong influence of the azimuthal magnetic field on the early development stage of shock formation. Conversely, the slower rate of decrease in shock thickness with increasing values of α suggest that the effect of the azimuthal magnetic field may eventually saturate. Overall, the results demonstrate that an increase in the azimuthal attractive magnetic field will produce a faster and sharper shock wave by focusing all available momentum and energy into a smaller volume of space behind the shock front.

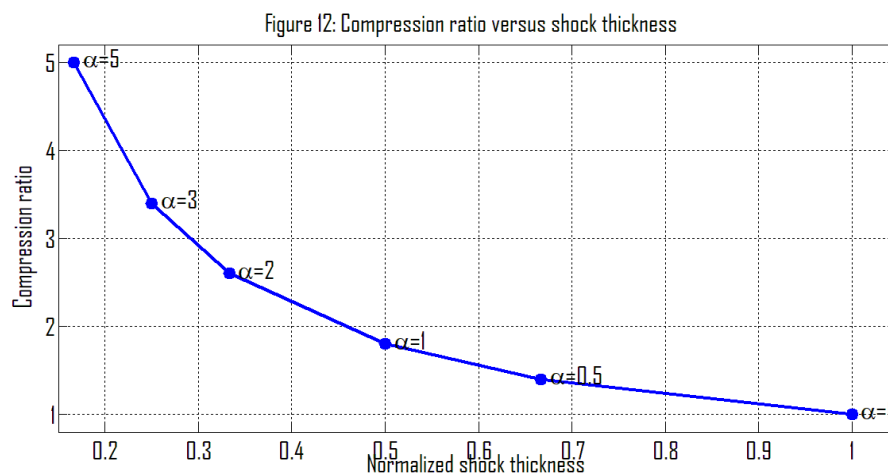
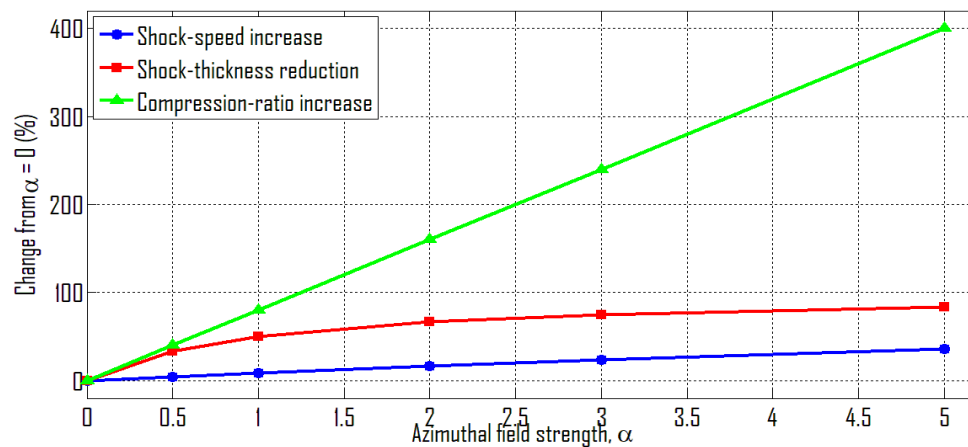


Figure 12 shows how the compression ratio depends on the normalized shock thickness for various strengths of the azimuthal magnetic field (α). From this plot we can see that there is an inversely proportional dependence between both variables; the compression ratio is very large when the shock thickness is small. The classical shock ($\alpha = 0$) has the largest normalized shock thickness (i.e., 1.0) but also the smallest compression ratio. When α increases, the magnetic field causes better rotationally convergent flow than would be present without it and thus produces better focusing of the fluid mass into the shock. Better focusing means greater nonlinear compression in the shock with a correspondingly smaller thickness. Thus, as the field gets stronger so does the compression ratio. While the rate of growth of the compression ratio increases most rapidly at intermediate values of α , it continues to grow as the field grows larger. Therefore, as shown in Fig. 12, stronger azimuthal attractive fields produce both thinner and more highly compressed shock waves through the effects described above.

Figure 13: Relative enhancement of shock properties



The relative enhancements in shock wave features (the three properties of shocks) are shown in Figure 13 as functions of increasing azimuthal magnetic field strength (α). As the strength of the azimuthal attractive field increases, so does each property; however, they increase at different rates. The enhancement of compression ratio has the largest rate of increase, increasing approximately linearly over a range of 0% to 400% for $\alpha = 5$. Thus, the azimuthal attractive field can be thought of as an intense enhancer of compressive shock wave strengths. Similarly, the rate of decrease in shock wave thickness continues to grow until it reaches 80% at the maximum value of α . This indicates that a larger degree of rotation will produce a smaller, more narrow shock front. Conversely, the rate of increase in shock wave speed is less rapid, resulting in an approximate increase of 35-40%. Therefore, while the azimuthal attractive field will accelerate shock waves through space, it has a greater effect upon shock intensity (i.e., compression) and shock sharpness (i.e., shock thickness) than upon the acceleration of the shock. Overall, this figure clearly illustrates that the primary function of the azimuthal attractive field is to create stronger, thinner shock waves through increased compression and decreased shock thickness, while creating a relatively small enhancement in shock speed.

Figure 14: Sensitivity of shock properties to field strength

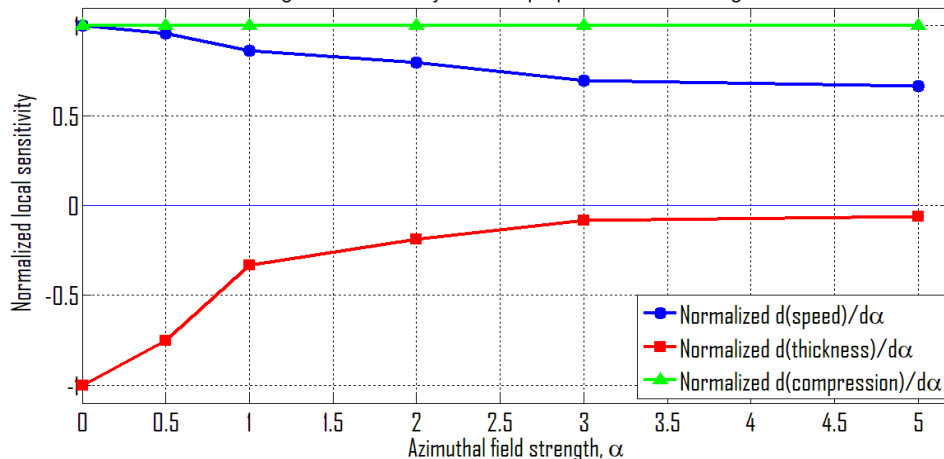


Figure 14 depicts the normalized localized sensitivity for shock speed, shock thickness, and compression ratio concerning azimuthal field strength (α) by comparing their respective derivative ratios. The comparison clearly displays the proportional relationship between each of the shock parameters' sensitivities. The normalized sensitivity for the compression ratio was found to remain relatively constant over all of the tested alpha ranges with an average value of roughly one. This indicates that the compression ratio is consistently the most sensitive of the three shock parameters and increases almost directly in relation to increased azimuthal field strength. Conversely, the normalized sensitivity for shock speed gradually decreases from around one down to about 0.65 as alpha increases. This indicates that, despite being accelerated by increased azimuthal field strength, the shock speed's rate of increase diminishes as it becomes increasingly larger. Lastly, the normalized sensitivity of the shock thickness is negative due to a decrease in shock thickness as alpha increases. Additionally, this sensitivity is largest when alpha is smallest; which means that there is initially a large effect of decreasing shock thickness as the azimuthal field strength is first introduced into the system. However, as alpha continues to increase, the normalized sensitivity begins to approach zero; which suggests that the rate of decreasing shock

thickness will begin to level off and eventually be limited. Overall, Figure 14 shows that the azimuthal attractive field has its greatest positive impact on compression enhancement and its ability to affect shock speed diminishes as a function of increased field strength. The effect of the azimuthal attractive field on shock thickness also becomes saturated at higher levels of field strength. Therefore, Figure 14 supports the idea that increasing the azimuthal attractive field primarily enhances shock compression while creating only minor additional increments of shock speed and shock-front thickness.

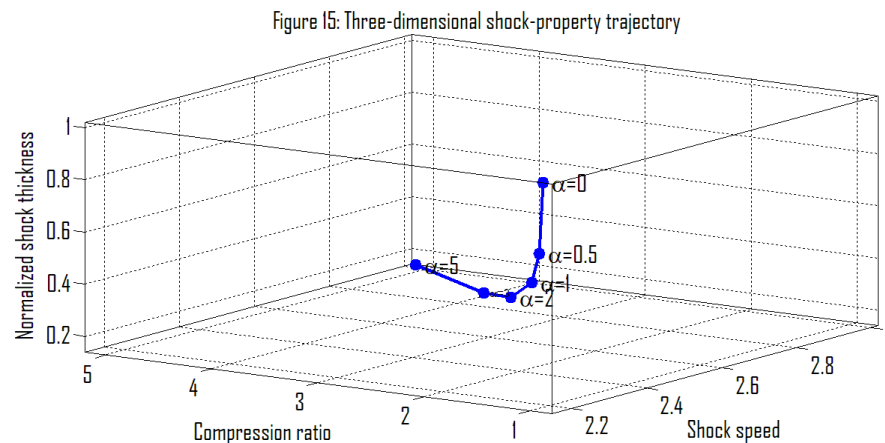


Figure 15 illustrates the three dimensional trajectory for shock wave property, displaying both the relationship between shock velocity, compression ration, and normalized shock thickness simultaneously, as function of the azimuthal magnetic field strength (α), for an ensemble of different values for α . Classical results are indicated by points labelled with $\alpha = 0$. Results corresponding to stronger fields were obtained using successively larger values of α . A clear trend exists; an increased attractive azimuthal field strength has the effect of increasing the shock speed and compression ratio, along with decreasing the normalised shock thickness. For $\alpha = 0$, the shock is less than one unit in thickness, travels at the lowest speed of all cases shown, and produces the least compressive shock; this represents the classical planar shock. The rotational converging effects induced by the azimuthal attractive field cause mass, momentum and energy to be concentrated into smaller regions of flow as α increases. This concentration of mass, momentum and energy have the effect of increasing the shock speed and compression while continuously reducing the shock thickness. The three dimensional display illustrates that the shock properties evolve together, rather than independently. In addition, it illustrates that their collective behavior follows a single physical path defined by increasing field strength. The curved nature of the display illustrates non-linear response, where the largest reductions in shock thickness occur at low values of α , while compression and shock speeds continue to increase at high values of α . It is obvious from Figure 15 that the azimuthal attractive field produces a transformation of the classical shock into a faster, thinner, and more highly compressed shock structure, which illustrates the very close interaction among the primary characteristic of shock waves.

10. Applications and Limitations: In addition to having theoretical application; the present study will have practical uses as well. One area in which there will be a great deal of potential use of this study is in designing future aerospace vehicles. For example, NASA's current X-30 (also known as the National Aerospace Plane) is being designed with the capability of hypersonic flight (i.e., speeds greater than Mach 5). At these speeds, shockwaves become a critical factor in vehicle design. If engineers could understand exactly how external magnetic fields affect the way shocks behave then they could potentially incorporate designs into their aircraft that would maximize performance while minimizing drag. Another area in which the present research will find use is in analyzing the motion of fluids in a rotating frame of reference. A good example of this is found in the study of rotating fluids in a turbine. Engineers who work with these types of systems need to know precisely how fluid behaves when moving through a region of space that is subject to a strong rotation. This knowledge will allow them to better predict flow characteristics in order to make improvements in efficiency and safety. Lastly, researchers working in the field of astrophysics may find some relevance in using this model. Astrophysicists study the properties of matter and radiation in extreme environments throughout the universe. Many of the models used by astrophysicists are based on highly simplified versions of the Navier-Stokes equations. However, many recent studies have shown that even small variations from spherical symmetry can lead to dramatic changes in global behavior. As such, studying how various external fields affect shockwave formation and propagation will aid in the development of new and improved models of astrophysical phenomena. While the study presented here provides much insight into shock wave behavior under various conditions, it is based on certain simplifications. First, we assume that the gas is non-conductive. We ignore electromagnetic interactions due to this assumption. Second, we assume that the flow is either inviscid or nearly inviscid. Viscosity causes dissipation and affects the boundary layers near

surfaces. Third, we neglect thermal conduction. Therefore, we do not account for heat transfer during the process. Fourth, our simple model for the azimuthal magnetic field does not accurately represent all possible real world force fields. Fifth, the values chosen for use in our illustration were arbitrary and therefore normalized, and did not come from any real experiments.

11. CONCLUSION

The current research has demonstrated that the application of an azimuthal attractive force changes the classical theory of shock-wave generation in nonconducting gases. In comparison to the traditional one dimensional planar shock, the presence of azimuthal forces causes rotational convergence, thus inducing multidimensional flow behaviors resulting in curved or spiral shock structures. The results have indicated that increased strengths of the azimuthal attractive force cause an increase in shock speeds, reduction in shock thicknesses and significant enhancement of shock compression ratios. As a result, shocks with greater and sharper characteristics are formed. These properties were however shown to be less stable under greater azimuthal attractive forces; therefore, there was an indication that larger degree of asymmetries and potentially distorted shock shapes could occur. In addition, the current study demonstrated that shock waves are no longer generated exclusively through longitudinal pressure gradient, but rather the radial momentum and concentrated energy of the azimuthal fields produce strong influences upon shock generation. Additionally, the numerical and dimensionless analyses confirmed the previous theoretical predictions and provided insight into the transitions between classical and field dominated shock regimes. While the model utilized several simplifications, it does provide a useful basis for the comprehension of the complicated shock generation processes occurring within rotating and field influenced environments.

REFERENCES

- [1]. Anand, R. K. (2025): "MHD Rankine–Hugoniot jump conditions for shock waves in van der Waals gases", *arXiv Preprint*, arXiv:2507.07564:1–15.
- [2]. Chauhan, S., Singh, D. (2025): "Analysis of magnetogasdynamic shock wave propagation in a rotational axisymmetric self-gravitating non-ideal gas", *European Physical Journal Plus*, 140(10):987–1003.
- [3]. King, L. E. (2025): "Flow discontinuities in compressible fluid systems", *Springer Fluid Mechanics Series*, 12(1):45–72.
- [4]. Kumar, R., Singh, U., Mishra, A. K. (2025): "Propagation of shock waves in non-ideal magnetohydrodynamic flows with thermal radiation and viscosity effects", *International Journal of Innovative Science and Research Technology*, 10(6):1814–1819.
- [5]. Ma, Q., Chong, K., Zhou, Q. (2025): "Strong shock propagation for the finite-source circular blast in a confined domain", *Applied Mathematics and Mechanics*, 46(6):891–910.
- [6]. Nath, G., Kadam, V. S. (2025): "Lie group theoretic method for magnetogasdynamic shock wave in a rotational axisymmetric real gas", *International Journal of Applied and Computational Mathematics*, 11(3):Article 135.
- [7]. Nath, G., Kadam, V. S. (2025): "Propagation of shock waves in weakly conducting non-ideal gas in the presence of azimuthal and axial magnetic inductions", *Journal of Engineering Physics and Thermophysics*, 98(4):1023–1035.
- [8]. Nath, G., Maurya, A. (2025): "Group invariance method for spherical shock wave in a non-ideal gas under the influence of gravitational and azimuthal magnetic fields", *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences*, 95(1):85–102.
- [9]. Nath, G., Maurya, A. (2025): "Optimal system of Lie sub-algebras and numerical solution for shock wave in rotating non-ideal dusty gas with radiation", *Journal of Engineering Mathematics*, 134(2):245–263.
- [10]. Rajput, A., Sharma, V. (2025): "Shock wave dynamics in non-ideal magnetogasdynamic medium with variable thermal conductivity", *JETIR Journal of Emerging Technologies*, 12(5):221–230.
- [11]. Singh, D. (2026): "Shock wave dynamics in a non-ideal rotating dusty gas with magnetic field and radiation", *International Journal of Engineering Science*, 182(2):103765.
- [12]. Singh, M., Arora, R. (2025): "Converging cylindrical shock waves in van der Waals gas with dust particles under the effect of magnetic field", *European Physical Journal Plus*, 140(5):456–472.
- [13]. Upadhyay, P., Nath, G. (2025): "Lie symmetry approach for shock propagation in self-gravitating non-ideal gas with radiation and magnetic field", *Indian Journal of Physics*, 99(6):1519–1540.
- [14]. Yadav, S. (2025): "Lie group invariance technique for analyzing strong shock waves in non-ideal gas with azimuthal magnetic field", *Mathematical Methods in the Applied Sciences*, 48(8):5120–5135.